

DERIVATIVE PRICING
A TECHNICAL GUIDE, PART II

DRAFT

Table of Contents

List of Abbreviations	4
Section 1 Introduction	5
Section 2 Hedging.....	6
2.1 Hedging objectives	6
2.2 Option Greeks	7
2.3 Delta hedging	8
2.4 Hedging FX risk.....	8
2.5 Hedging bond default risk	8
2.6 Accounting considerations	8
2.7 Examples of hedging strategies	8
2.7.1 Covered call	8
2.7.2 Married put	8
2.7.3 Protective collar	8
2.7.4 Bull call spread	9
2.7.5 Bear put spread	9
2.7.6 Long straddle.....	10
2.7.7 Long strangle.....	10
2.7.8 Risk reversal (RR) strategy	10
2.7.9 Butterfly (BF) spread strategy.....	10
2.8 Comparing derivatives and insurance contracts	11
Section 3 Summary.....	12
Appendix A Bloomberg Tools.....	13
A.1 OVML tool	13
A.1.1 Commodity price options	13
A.1.2 FX options	14
A.2 OVME tool for stock price options.....	16
A.3 OVCV tool	16
A.4 OAS1 tool	19
A.5 Implied volatility	23
A.6 SDRV tool.....	25
Appendix B Implementation of Lease Residual Value Risk Model.....	27
B.1 Modelling background	27
B.2 Data.....	27
B.2.1 Data request.....	27
B.2.2 Lease insurance contract	28
B.2.3 Data review and transformation.....	29

B.3	Modelling approaches	30
B.4	Estimation of model parameters	30
B.4.1	Approach	30
B.4.2	Implementation	31
B.5	Residual risk valuation for individual contracts	33
B.5.1	CDS approach	33
B.5.2	Black-Scholes approach	35
B.5.3	Mixed CDS / Black-Scholes approach	37
B.5.4	Summary of the residual risk valuation modelling	37
B.6	Residual risk valuation for a portfolio of contracts	39
Appendix C	ac.finance.* option pricing tools	40
C.1	ac.finance.BSM	40
C.2	ac.finance.APM	40
C.3	Overview	40
C.4	Model inputs	41
Appendix D	Derivative Markets Overview	42
D.1	Risk transfer process	42
D.2	Early termination of hedging contracts	43
Appendix E	Examples	45
E.1	Deal contingent FX hedging	45
E.1.1	FX hedging timing	45
E.1.2	FX hedging strategy design	45
E.1.3	Valuation of a DC forward	47
E.2	Book value hedging	48
Appendix F	Technical Comments	49
Appendix G	References	50
G.1	Lease valuation and hedging	50
G.2	Derivative pricing	50

List of Abbreviations

The following abbreviations and symbols are used in this guide:

AAF	Annuity adjustment factor
ac.APM	The option pricing tool described in these notes
ac.SRM	The interest rate derivative pricing tool
APM	Asset price model
ATM	At-the-money, the situation when the price of the underlying security equals the strike price
ATMF	At-the-money forward option, the situation when the price of the underlying security equals the option strike price on the forward market
bps	Basis point (same as pip, equal 1/100 th of 1%)
BS	Black-Scholes
BSM	Black-Scholes-Merton
BV	Book Value
CCY	Currency
CDS	Credit default swap
DC	Deal contingent
DTCC	Depository Trust And Clearing Corporation
ES	Expected shortfall
FMV	Fair market value
FV	Forward value
FXIP	FX Information Portal Bloomberg function
G/L	Gain and loss
M&A	Merger and acquisition
MTM	Marked-to-market
OCI	Other comprehensive income
OECD Guidelines	"BEPS Actions 8 – 10, Financial Transactions", a draft published in July – September 2018 for the purposes of public discussion
OV	Bloomberg option valuation tool
P&L	Profit and loss
Pip	Percentage in point (same as bps, equal 1/100 th of 1%)
PP	Purchase Price
PPE	Purchase price equation
SRM	Short rate model of interest rates
VaR	Value-at-risk

Section 1 Introduction

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Section 2 Hedging

The derivative prices are derived assuming market completeness by replicating the derivative instrument payoff structure. A derivative instrument is often used to hedge the risks of the underlying securities. Alternatively, underlying securities can be added to a derivative instrument to offset movements in the derivative price movement.

2.1 Hedging objectives

The first step in the design and implementation of a hedging strategy is to determine the objective of hedging. The objective is typically set as risk minimization. Therefore, the first step is to define and measure risk exposure.

One standard measure of risk that is traditionally applied in financial industry is value-at-risk (**VaR**) measure. VaR can be selected at different levels (e.g. at one or two standard deviations or at 'worst loss' level, where the VaR levels are estimated based on historical data). Formally, VaR metrics does not satisfy the definition of risk measure (see [8]). A general risk measure is modelled using **spectral risk** measure, which is defined as

$$(2.1) \quad M(X) = - \int_0^1 \theta(p) F_X^{-1}(p) dp$$

where $F_X^{-1}(p)$ is the inverse of cumulative distribution function of factor X and $\theta(p)$ is the spectral function which assigns weights to different outcomes of X . The metrics satisfies the definition of risk if function $\theta(p)$ satisfies the following properties:¹

1. Function $\theta(p)$ is non-negative: $\theta(p) \geq 0$;
2. Function $\theta(p)$ is normalized to one: $\int_0^1 \theta(p) dp = 1$
3. Function $\theta(p)$ is non-increasing: $\theta(p) \downarrow_p$

Note that VaR metrics does not satisfy property 3 from the above list. The modification of VaR metrics which satisfies the above properties is the expected shortfall (**ES**) metrics. The spectral function $\theta(p)$ for VaR and ES is described below:

$$(2.2) \quad \begin{cases} VaR: & \theta(p = \alpha) = 1 \\ ES: & \theta(p \leq \alpha) = \frac{1}{\alpha} \end{cases}$$

where α is the threshold probability value. The weights assigned to all other scenarios are equal to zero. Effectively, VaR metrics assigns full weight to the scenario which corresponds to the threshold value α , while the ES metrics assigns equal weights to all m scenarios which correspond to the threshold value α or below.

¹ https://en.wikipedia.org/wiki/Spectral_risk_measure.

2.2 Option Greeks

Greeks measure option sensitivity to different parameters. Greeks are typically applied for the following purposes:

1. Assess exposure to model parameters (such as underlying asset price, price volatility, time, risk-free rate, etc.);
2. Greeks are used as a convenient metrics to standardize market terminology such as bid/ask spread (measured in terms of volatility spreads) or difference between spot and strike price (measured in terms of option delta).

Exhibit below summarizes option Greeks.

Exhibit 2.1 Summary of option Greeks

Greek name	Option parameter	Hedging strategy
Delta (δ), Gamma (Γ)	Spot price	Option exposure to asset price can be covered by the purchase of the underlying asset
Vega (v)	Volatility	
Rho	Discount (risk-free) rate	Exposure to interest rate can be hedged by purchase of interest rate futures
Theta	Remaining maturity term	

The Greeks are illustrated selectively in the exhibits below.

Exhibit 2.2 Delta and Vega Greeks illustrated for a FX call option (GBP/CAD)

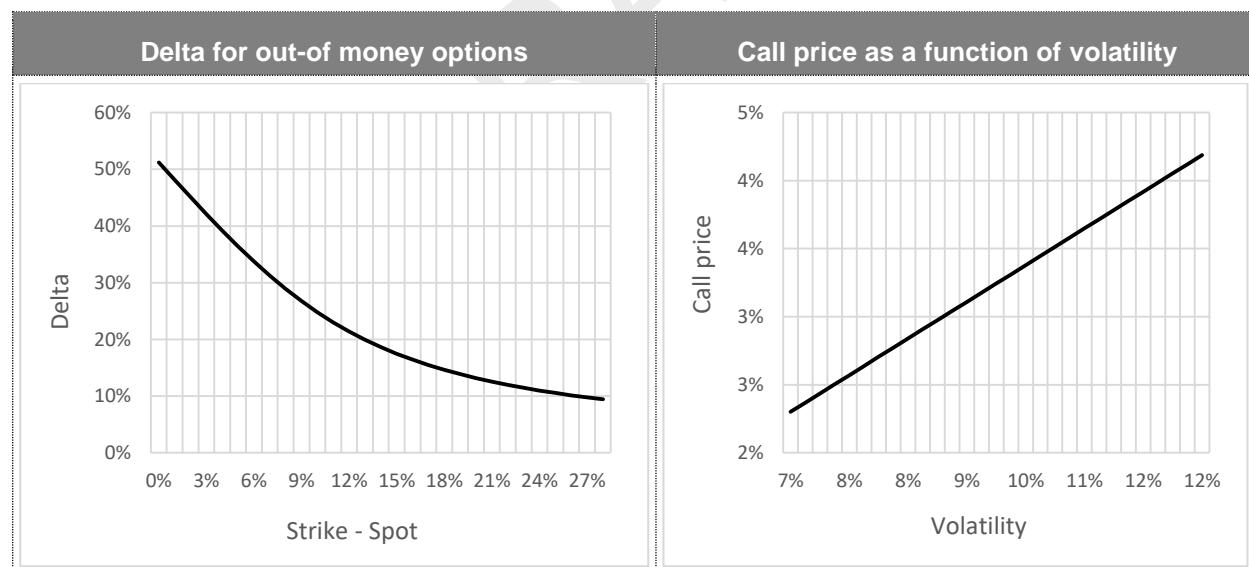


Exhibit above show that delta for the ATM option equals 0.5. The bid/ask spreads for out-of-the-money options are typically quoted in terms of delta: e.g. 25D refers to the option with difference between strike and spot corresponding to $\delta = 0.25$ (the lower is delta, the deeper is the option out-of-the-money). A standard practice in financial industry is to describe out-of-the-money options in terms of delta (see examples in Appendix A.5).

The right panel of the exhibit above illustrates that v is approximately constant for the option (in the example, $v = 33.69\%$). A standard practice in financial industry is to quote premium spreads on options in terms of volatility spreads

(see examples in Appendix A.5). For example, a 1% vol premium on a call option is estimated as follows: (i) get the mid-market option price; (ii) apply BSM equation to estimate the implied volatility that corresponds to the mid-market price; (iii) add a 1% premium to the implied volatility and recalculate the option price using BSM equations. In the example above, the call price equals 2.6% of the notional amount. A 1% vol premium corresponds to 0.34% premium on call option price.

2.3 Delta hedging

2.4 Hedging FX risk

Net exposure.

2.5 Hedging bond default risk

2.6 Accounting considerations

Fair value hedge.

Cash flow hedge.

Hedging a portion of risk component.

2.7 Examples of hedging strategies

A list of some frequently used hedging strategies is presented below.²

2.7.1 Covered call

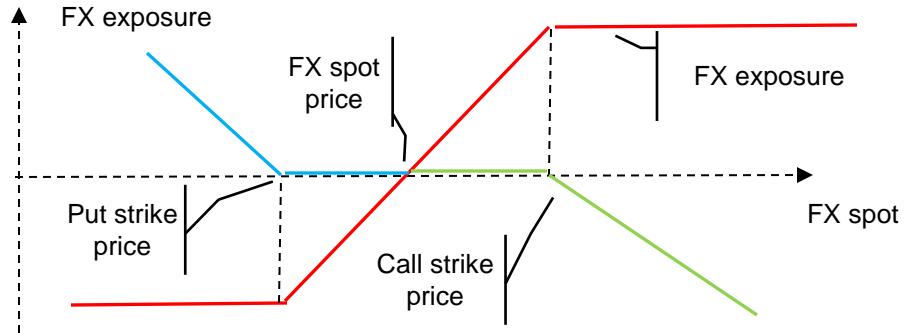
2.7.2 Married put

2.7.3 Protective collar

A collar strategy that is constructed by (i) holding shares of the underlying stock, (ii) buying a put option to protect against the downside risk; and (iii) sell call option to reduce the hedging price by limiting the upside gain. The exposure in the collar strategy is illustrated in the diagram below.

² <https://www.investopedia.com/trading/options-strategies/>

Exhibit 2.3 'Collar' hedging strategy

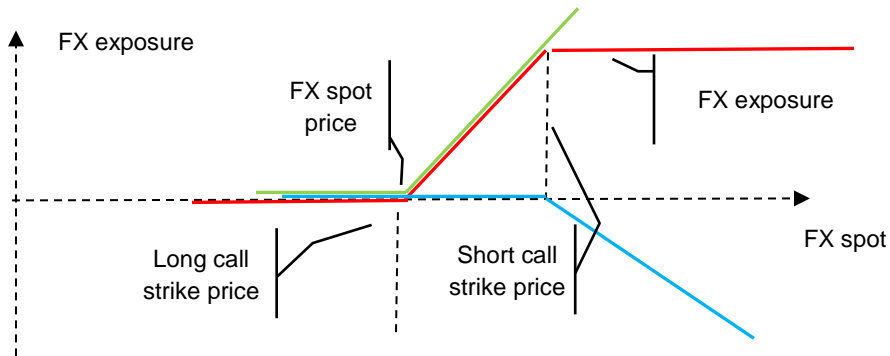


The payout in the call and put option contracts are shown by the blue line.

2.7.4 Bull call spread

The 'bull call spread' strategy provides limited protection of the asset price by hedging the price movement above the 'short call strike price'. The buyer has price exposure within the [long call strike, short call strike] range. Limited protection is compensated by lower cost of the hedging strategy.

Exhibit 2.4 'Bull call spread' hedging strategy

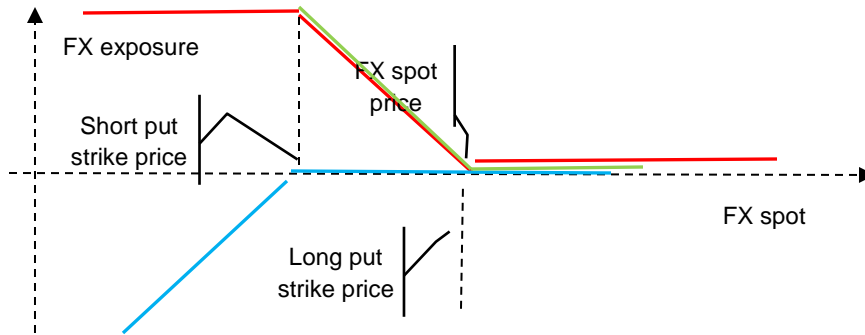


If compared to an insurance policy, 'bull call spread' strategy can be viewed as an insurance against upward price movement with the cap on the total payout under the insurance policy.

2.7.5 Bear put spread

The 'bear put spread' strategy provides limited protection of the asset price by hedging the price movement within the [long put strike, short put strike] range of price movement. Limited protection is compensated by lower cost of the hedging strategy.

Exhibit 2.5 'Bear put spread' hedging strategy



If compared to an insurance policy, 'bull call spread' strategy can be viewed as an insurance against downward price movement with the cap on the total payout under the insurance policy.

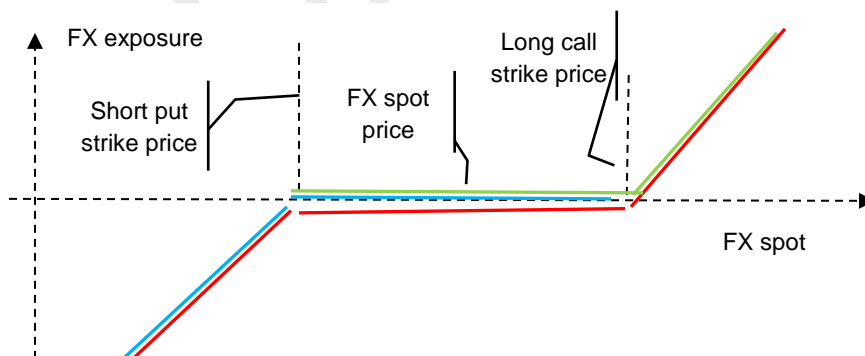
2.7.6 Long straddle

2.7.7 Long strangle

2.7.8 Risk reversal (RR) strategy

Risk reversal is a bullish strategy which involves (i) buying an out-of-the-money call option and (ii) selling an out-of-the-money put option. The buyer of the strategy bets on a large increase in the asset price and loses money if there is a large drop in the asset price.

Exhibit 2.6 'Risk reversal' hedging strategy



Bid/ask spreads on risk reversal option strategy are quoted in Bloomberg FXIP tool.

2.7.9 Butterfly (BF) spread strategy

Under the butterfly spread strategy the option buyer gains profit if the deviation from the spot price is small (effectively the option buyer bets on the price stability). The buyer of the BF strategy is protected from large deviations in the underlying asset price.

Exhibit 2.7 'Butterfly spread' hedging strategy

Bid/ask spreads on butterfly spread option strategy are quoted in Bloomberg FXIP tool.

2.8 Comparing derivatives and insurance contracts

Premium

Payout

Deductible – strike price

Cap on payout

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Section 3 Summary

This section presents the summary of the key results and equations which are spread out across this guide. The section also provides a link to the sections where the equations are derived and discussed in more detail.

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Appendix A Bloomberg Tools

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A.1 OVML tool

OVML tool is applied for FX and commodity price option valuation.

A.1.1 Commodity price options

This section describes Bloomberg's OV valuation tool applied to commodity price options and forwards. The tool is illustrated for the gold commodity. Bloomberg print screen with the OV tool output is illustrated below.

Exhibit A.1 Bloomberg OV tool output for the commodity price option

Field	Value
Price Date	03/23/21 13:59
Asset	XAUUSD
Product	Gold
Source	OTC
Spot	Mid 1727.93
Style	European Vanilla
Direction	Client buys Physical
Call/Put	XAU Call
Expiry	1 year 03/23/22
Delivery	NY 09:30 03/25/22
Strike	1733.36 ATMF
Notional	XAU 100.00
Converted	troy ounce 100.00
Model	Black-Scholes
Vol	TPMT 17.399%/17.399%
Vol Spread	0.000%
Points	BGN Mid 543.10
Forward	Mid 1733.36...
Contango Rate	0.31...
XAU Depo	Implied Mid -0.273...%
USD Depo	USD SOFR Mid 0.040%
Gamma	XAU 2.29
Vega	0.40
Price	% XAU 6.9515% P
Premium	XAU 6.95 P
Prem Date	03/25/21
Delta	Spot 53.6126%
Hedge	-53.61

The tool uses the following default options:

1. The strike price is set equal to the gold futures price (ATMF – at the money forward price).

Dates	1W	1M	2M	3M	6M	9M	12M	2Y
T 3/23/21	5.7100	28.8000	65.4800	99.3900	225.9300	354.2700	543.2200	970.0200
M 3/22/21	8.4537	26.0852	59.8998	92.3140	221.8221	349.8705	555.3953	943.5244

The gold forward points are available through the 'FXXUUS01 Index' series or, alternatively through the 'XAUSR12M Index' series which presents forward points in bps of the gold spot price.

Date	Last Price
Fr 03/26/21	
Th 03/25/21	
We 03/24/21	
Tu 03/23/21	.3100
Mo 03/22/21	.3150

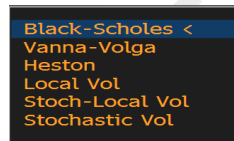
2. Volatility is linked to Bloomberg volatility series, such as 'XAUUSDV1Y TPMT Curncy'³ shown below (the series represents XAUUSD 1 year ATM implied volatility).

Date	Last Price
Fr 03/26/21	
Th 03/25/21	
We 03/24/21	
Tu 03/23/21	17.4000
Mo 03/22/21	17.7000

3. Discount rate is set at USD deposit rate. The rates are defined as 'USD SOFR overnight index swap rates'⁴ which are the swap rates constructed for the SOFRRATE Index⁵ curve.

Dates	9M	10M	11M	12M
T 3/23/21	0.0350	0.0378	0.0400	0.0404
M 3/22/21	0.0300	0.0337	0.0362	0.0390

4. Model. The list of models that can be selected for the option estimation is shown below. The default model is Black-Scholes.



The results of the Bloomberg commodity option price test are shown in the exhibit below.

Exhibit A.2 Test results for the Bloomberg commodity price option tool

A.1.2 FX options

This section describes Bloomberg's OV valuation tool applied to FX options and forwards. The tool is illustrated for the GBP/CAD currency pair. Bloomberg print screen with the OV tool output is illustrated below. (Bloomberg screen for the OV tool can be alternatively presented using Bloomberg's **OVML** tool).

³ For consistency, we select the same source TPMT as in the option valuation tool.

⁴ The swap rates are reported through the YCSW0490 USD SOFR (vs. Fixed Rate) Bloomberg curve.

⁵ SOFRRATE (Secured Overnight Financing Rate) is the index for the broad measure for the cost of borrowing cash overnight collateralized by Treasury securities.

Exhibit A.3 Bloomberg OV tool output for the FX option

31) Solver (Premium) ▾		32) Load	
61) Deal 1		62) +	
51) Pricing		53) Scenario	
OVML GBPCAD EU 1.7085C 12/16/20 N1M			
Strategy 1 ▾		Leg 1 ▾	
Price Date		09/16/20	14:12
Asset		GBPCAD	
Spot	Mid ▾	1.7081	
Style	European ▾	Vanilla ▾	
Direction	Client buys ▾	Physical ▾	
Call/Put	GBP ▾	Call ▾	
Expiry	3 months ▾	12/16/20	
Delivery	NY 10:00 ▾	12/18/20	
Strike	1.7085	ATMF	
Notional	GBP ▾	1,000,000.00	
Model	Black-Scholes ▾		
More Market Data			
Vol	BGN	8.963%/10.447%	
Vol Spread		1.484%	
Points	Cross	Mid ▾	3.59
Forward		Mid ▾	1.7085...
GBP Depo	GBP vs.	Mid ▾	0.094...%
CAD Depo	Implied	Mid ▾	0.178...%
Greeks			
Gamma	GBP ▾	82,285.19	
Vega		1,990.93	
Results			
Price	% GBP ▾	2.0802% P	
Premium	GBP ▾	20,802.15 P	
Prem Date		09/18/20	
Delta	Spot ▾	50.9545%	
Hedge		-509,545.30	

The tool uses the following default options:

1. The strike price is set equal to the futures price

GBPCAD3M BGN Currency		96 Expo	
GBP-CAD X-RATE		3 MO	
Range	09/16/2019	-	09/16/2020
Market	Last Price		
View	Price Table		
Date	Last Price		
Fr 09/18/20			
Th 09/17/20			
We 09/16/20	1.708771		
Tu 09/15/20	1.699985		
Mo 09/14/20	1.692975		

2. GBP risk-free rate. Bloomberg constructs a custom curve for the GBP risk-free rate. Other options include (i) GBP overnight rate (BP000/N ticker); (ii) Overnight Index Swap (OIS) rate (which swaps floating overnight rate into equivalent fixed rate with given maturity), e.g. BPSWSC for a 3-month swap rate
3. CAD risk-free rate. Bloomberg constructs a custom curve for the CAD risk-free rate. Other options include (i) Canadian overnight Repo rate (CAONREPO ticker); (ii) Overnight Index Swap (OIS) rate, e.g. CDSOC for a 3-month swap rate.
4. The price is quoted as a percentage of spot price.

The Bloomberg print screen with the OV calculation results was replicated using equation **Error! Reference source not found..** The results are shown in the exhibit below.

Exhibit A.4 Test results for the Bloomberg FX option tool

Parameters							Option calculation		
#	T	S	K	sigma	r	d	call	call (% of spot)	implied volatility
Bloomberg OV tool test									
1	0.25	1.7081	1.7085	8.963%	0.178%	0.094%	0.031	1.79%	8.96%
2	0.25	1.7081	1.7085	10.447%	0.178%	0.094%	0.036	2.08%	10.45%

The 2.08% option price is replicated using 10.447% volatility (upper bound of the volatility range) and by dividing the calculated call option price by FX spot price.

A.2 OVME tool for stock price options

This section describes Bloomberg's OV valuation tool applied to stock price options and forwards. The tool is illustrated for the 'GOLD US Equity' stock ticker (Barrick Gold Corporation). Bloomberg print screen with the OV tool output is illustrated below.

Exhibit A.5 Bloomberg OV tool output for the stock price option

31) Pricing	32) Scenario	33) Matrix	34) Volatility	35) Backtest			
Underlying	GOLD US Equity	BARRICK GOLD CRP	Trade	03/23/2021 14:37			
Und. Price	Mid	20.165 USD	Settle	03/23/2021			
Results							
Price (Total)	1.37	Currency	USD	Vega	0.04	Time Value	1.37
Price (Share)	1.3748	Delta (%)	53.10	Theta	0.01	Gearing	14.67
Price (%)	6.8179	Gamma (%)	2.3174	Rho	0.00	Break-Even (%)	6.82
American Vanilla Leg 1							
Style	Vanilla						
Exercise	American						
Call/Put	Call						
Direction	Buy						
Strike	20.165						
Strike % Money	ATM						
Shares	1.00						
Expiry	06/21/2021 16:30						
Time to Expiry	90 01:53						
Model	BS - disc.						
Vol	BVOL	Ask	35.199%				
Forward	Carry		20.0844				
USD Rate	MMkt		0.191%				
Dividend Yield			1.818%				
Discounted Div Flow			0.09				

The strike price is set equal to the stock spot price (20.165). The yield rate proxy for the stock can be obtained through the DVD Bloomberg function.

A.3 OVCV tool

In this section we present the results of convertible bond valuation performed for the bond illustrated in Sections **Error! Reference source not found.** and **Error! Reference source not found.**. Bloomberg print screen with the convertible bond valuation output is shown below.

Exhibit A.6 Bloomberg OVCV tool output

LUV 1 05/01/25		91) Actions		92) Settings		Convertible Valuation			
Bond B1792727		Stock LUV US Equity							
11) Pricing Analysis		12) Cash Tender		13) Historical Analysis		14) Scenario Analysis		15) Nuke/Hedge	
20) Analysis		22) IR Curve		23) Credit Curve		24) Dividends		25) Volatility	
Market Price	121.200	Spread (OAS)	194.885	Volatility	58.59	Stock Price	34,360	Borrow Cost	0.0 %
Historical (06/...		Flat 5 Year Spre...		Flat 1Y Implied					
Trade Date	06/22/2020	Settle Date	06/24/2020	Model	Black-Scholes	E2C	0.0	Greeks based on	Mkt Price & Vol
Fair Value	138.029	Bond Floor	95.090	IR Sens	-2.385	Yield to Mty	-2.798		
Cheapness (%)	12.192	Option Value	42.939	Spread Sens	-2.263	Yield to Call	N.A.		
Implied Spread	800.140	Parity	89.305	Phi	-0.191	Yield to Put	N.A.		
Implied Vol	34.104	Premium (pts)	31.895	Psi	-2.324	Yield to Worst	-2.798		
Delta (%)	80.834	Premium (%)	35.715	Chi	N.A.	Current Yield	1.031		
Delta (pts)	0.722	Gamma	0.211	Upsilon	0.000	Breakeven (Y)	25.516		
Effective Trig	0%/0.000	Vega	0.536	Convexity	-0.335	CF Payback (Y)	25.516		
Unit Prc	1.380M	Theta	0.002	Effective Dur	1.968	Accrued Int	0.184		
Hedge Ratio	2.101	Exp Life (Fugi...	4.850						
Description									
Bond CUR	USD	Conv Prc	38.4750	Issue Amt	2.30MMM	Next Call Date	None		
Stock CUR	USD	Conv Ratio	25.9909	Amt Out	2.30MMM	Next Put Date	None		
Stock Ticker	LUV US	Proj Conv Ratio	25.9909	Issue Date	05/01/20	Next Call Price	None		
Cusip	844741BG2	Init Prm (%)	35.00	Maturity	05/01/25	Next Put Price	None		
Collateral	SR UNSECURED	Coupon	1.25% FIXED	Conv From	07/01/20	Prov Trig	None		
Par Amount	1000.00	Cpn Freq	Semi-Annual	Conv Until	04/30/25	Prov Start	None		

The parameters of the OVCV tool are interpreted as follows:

1. **Market Price:** price of the convertible bond (as shown by Bloomberg HP function).
2. **Spread (Credit):** bond risk premium. The spread is added to risk-free rate to discount projected cash flows in Black-Scholes model.
3. **Volatility:** stock price volatility. Parameter can be set either based on historical or implied volatility.
4. **Implied volatility:** the value of volatility which produced the bond fair value equal to the quoted market value.
5. **Cheapness:** difference between the model and the market price expresses as a percentage of model price.
6. **Bond floor:** NPV of fixed income components
7. **Option value:** difference between the bond fair value and bond floor
8. **Yield to Maturity (YTM):** the YTM value is estimated based on the bond market price.

The valuation convertibility option value is estimated based on the estimated historical volatility and estimated risk spread. As a result, the bond FMV value 134.550 does not match the market bond price 121.728. The convertibility option is valued based on the estimated FMV value ($39.274 = 134.550 - 95.276$).⁶

Alternatively, spread estimate can be replaced with the implied spread or volatility to ensure consistency between the bond market price and the FMV.

⁶ The yield rates (e.g. yield to maturity = -2.888) are reported based on the market price of 121.728.

Adjust 'Spread (OAS)' so that 'Market Price' equals 'Fair Value'

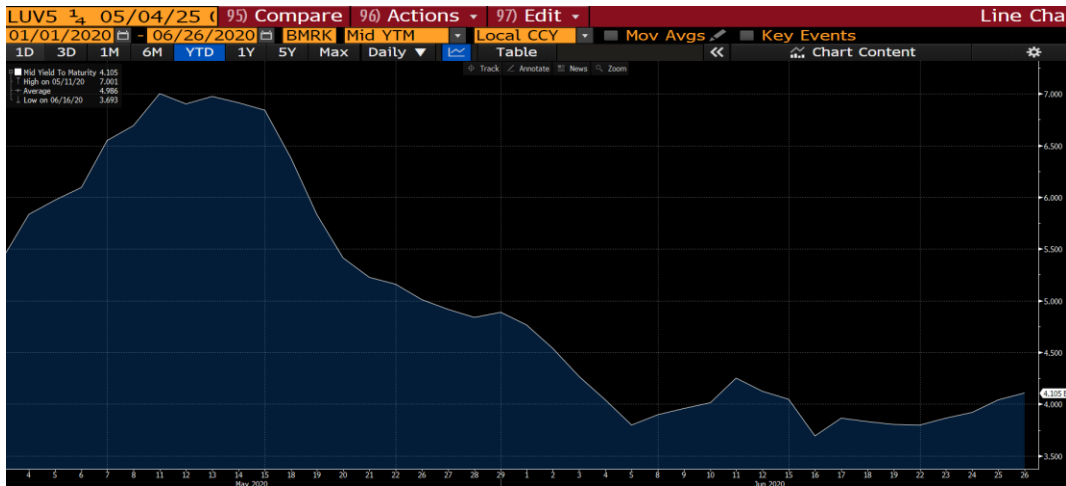
Convertible Valuation							
Bond BJJ1792727 Stock LUV US equity							
1) Pricing Analysis 2) Cash Tender 3) Historical Analysis 4) Scenario Analysis 5) Nuke/Hedge							
2) Analysis 2) IR Curve 2) Credit Curve 2) Dividends 2) Volatility							
Market Price	Spread (OAS)	Volatility	Stock Price	Borrow Cost			
121.200	800.000	58.59	34.360	0.0 %			
Historical (06/... User Flat Credit Flat 1Y Implied)							
Trade Date	Settle Date	Model	E2C	Greeks based on			
06/22/2020	06/24/2020	Black-Scholes	0.0	Mkt Price & Vol			
Fair Value	121.200	Bond Floor	71.530	IR Sens	-2.385	Yield to Mty	-2.798
Cheapness (%)	0.000	Option Value	49.670	Spread Sens	-2.263	Yield to Call	N.A.
Implied Spread	800.140	Parity	89.305	Phi	-0.191	Yield to Put	N.A.
Implied Vol	58.592	Premium (pts)	31.895	Psi	-2.324	Yield to Worst	-2.798
Delta (%)	80.834	Premium (%)	35.715	Chi	N.A.	Current Yield	1.031
Delta (pts)	0.722	Gamma	0.211	Upsilon	0.000	Breakeven (Y)	25.516
Effective Trig	0%/0.000	Vega	0.536	Convexity	-0.335	CF Payback (Y)	25.516
Unit Prc	1.212M	Theta	0.002	Effective Dur	1.968	Accrued Int	0.184
Hedge Ratio	2.101	Exp Life (Fugi...	4.850				
Description							
Bond CUR	USD	Conv Prc	38.4750	Issue Amt	2.30MMM	Next Call Date	None
Stock CUR	USD	Conv Ratio	25.9909	Amt Out	2.30MMM	Next Put Date	None
Stock Ticker	LUV US	Proj Conv Ratio	25.9909	Issue Date	05/01/20	Next Call Price	None
Cusip	844741BG2	Init Prm (%)	35.00	Maturity	05/01/25	Next Put Price	None
Collateral	SR UNSECURED	Coupon	1.25% FIXED	Conv From	07/01/20	Prov Trig	None
Par Amount	1000.00	Cpn Freq	Semi-Annual	Conv Until	04/30/25	Prov Start	None

The example shows that implied spread generates in this case a significantly higher adjustment for bond convertibility option.

The yields on a comparable non-convertible bond issued by the same company are shown in the exhibits below.

LUV 5 05/04/25 \$1104.946 -0.320 381.3 bps vs 1.0250 05/31/2025							
At 12:58 Source BMRK							
LUV 5 05/04/25 Corp Settings Actions Page 1/12 Security Description: Bond							
9) No Notes 9) Buy 9) Sell							
2) Bond Description 2) Issuer Description							
Pages		Issuer Information			Identifiers		
1) Bond Info	Name SOUTHWEST AIRLINES CO			ID Number	BJ1961538		
2) Addtl Info	Industry Airlines (BCLASS)			CUSIP	844741BJ6		
3) Reg/Tax	Security Information			ISIN	US844741BJ60		
4) Covenants	Mkt Iss Global			Bond Ratings			
5) Guarantors	Country	US	Currency	USD	Moody's	Baa1	
6) Bond Ratings	Rank	Sr Unsecured	Series		S&P	BBB *	
7) Identifiers	Coupon	5.250000	Type	Fixed	Fitch	BBB+	
8) Exchanges	Cpn Freq	S/A			Composite	BBB	
9) Inv Parties	Day Cnt	30/360	Iss Price	99.78300	Issuance & Trading		
10) Fees, Restrict	Maturity	05/04/2025			Amt Issued/Outstanding		
11) Schedules	MAKE WHOLE @50.000000 until 04/04/25/ CALL ...			USD	1,250,000.00 (M) /		
12) Coupons	Iss Yield	5.300			USD	1,250,000.00 (M)	
Quick Links	Calc Type	(1)STREET CONVENTION			Min Piece/Increment		
3) ALLQ Pricing	Pricing Date	04/29/2020			2,000.00 / 1,000.00		
3) QRD Qt Recap	Interest Accrual Date	05/04/2020			Par Amount		
3) TDH Trade Hist	1st Settle Date	05/04/2020			1,000.00		
3) CACS Corp Action	1st Coupon Date	11/04/2020			Book Runner		
3) CF Prospectus				JOINT LEADS			
3) CN Sec News				Reporting			
3) HDS Holders				TRACE			
6) Send Bond							

The yield rates on the non-convertible bond are shown in the exhibit below.



A.4 OAS1 tool

25) Bond Description		26) Issuer Description	
Pages 11) Bond Info 12) Addtl Info 13) Reg/Tax 14) Covenants 15) Guarantors 16) Bond Ratings 17) Identifiers 18) Exchanges 19) Inv Parties 20) Fees, Restrict 21) Schedules 22) Coupons Quick Links 32) ALLQ Pricing 33) QRD Qt Recap 34) TDH Trade Hist 35) CACS Corp Action 36) CF Prospectus 37) CN Sec News 38) HDS Holders 60) Send Bond		Issuer Information Name DOW CHEMICAL CO/THE Industry Chemicals (BCLASS) Security Information Mkt Iss Domestic MTN Country US Currency USD Rank Sr Unsecured Series NOTZ Coupon 3.150000 Type Fixed Cpn Freq S/A Day Cnt 30/360 Iss Price 100.00000 Maturity 08/15/2029 CALL 07/07/20@100.00 Iss Yield 3.150 Calc Type (1)STREET CONVENTION Pricing Date 08/05/2019 Interest Accrual Date 08/08/2019 1st Settle Date 08/08/2019 1st Coupon Date 02/15/2020	
DOW 3.15 08/15/29 \$100.044 - .001 100.042 / 100.047 .967 / .738 At 11:15 --x-- Source BVAL		Identifiers ID Number AZ8672877 CUSIP 26054LJ79 ISIN US26054LJ799 Bond Ratings Moody's Baa2 S&P BBB- Fitch BBB+ Composite BBB Issuance & Trading Amt Issued/Outstanding USD 9,659.00 (M) / USD 9,659.00 (M) Min Piece/Increment 1,000.00 / 1,000.00 Par Amount 1,000.00 Book Runner INCAP-sole Reporting TRACE	

DOW 3.15 08/15/29 Corp Settings Actions Page 11/12 Security Description: Bond

94) No Notes 95) Buy 96) Sell

25) Bond Description 26) Issuer Description

Pages

- 11) Bond Info
- 12) Addtl Info
- 13) Reg/Tax
- 14) Covenants
- 15) Guarantors
- 16) Bond Ratings
- 17) Identifiers
- 18) Exchanges
- 19) Inv Parties
- 20) Fees, Restrict
- 21) Schedules
- 22) Coupons

Quick Links

- 32) ALLQ Pricing
- 33) QRD Qt Recap
- 34) TDH Trade Hist
- 35) CACS Corp Action
- 36) CF Prospectus
- 37) CN Sec News
- 38) HDS Holders

60) Send Bond

Schedules

Call Schedule

Call with minimum 10 days notice

May be called in full (only)

Callable on and anytime after date(s) shown

Date	Price
02/15/2020	100.000

DOW 3.15 08/15/29 \$1100.044 - .001 100.042 / 100.047 .967 / .738

At 11:15 -- x -- Source BVAL

AZ867287 Corp 96) Export 97) Settings Page 1/6 Historical Price Table

DOW 3.15 08/15/29

Range 08/05/2019 - 06/26/2020 Period Daily High 101.045 on 05/05/20

Market Mid Line Mid YTM Currency USD Average 95.466 on 03/23/20

View Price Table Source BVAL Net Chg .269 3.177 0.27%

Date	Mid Line	Mid YTM	Date	Mid Line	Mid YTM	Date	Mid Line	Mid YTM
Fr 06/26/20	100.044	3.144	Fr 06/05/20	100.041	3.144	Fr 05/15/20	101.013	3.023
Th 06/25/20	100.046	3.144	Th 06/04/20	100.044	3.144	Th 05/14/20	101.003	3.024
We 06/24/20	100.064	3.142	We 06/03/20	100.071	3.141	We 05/13/20	100.966	3.029
Tu 06/23/20	100.069	3.141	Tu 06/02/20	100.076	3.140	Tu 05/12/20	100.918	3.035
Mo 06/22/20	100.071	3.141	Mo 06/01/20	100.058	3.142	Mo 05/11/20	100.885	3.039
Fr 06/19/20	100.045	3.144	Fr 05/29/20	100.040	3.145	Fr 05/08/20	100.952	3.031
Th 06/18/20	100.045	3.144	Th 05/28/20	100.042	3.144	Th 05/07/20	100.994	3.026
We 06/17/20	100.061	3.142	We 05/27/20	100.053	3.143	We 05/06/20	100.998	3.025
Tu 06/16/20	100.062	3.142	Tu 05/26/20	100.066	3.141	Tu 05/05/20	101.045	3.020
Mo 06/15/20	100.047	3.144	Mo 05/25/20			Mo 05/04/20	100.992	3.026
Fr 06/12/20	100.039	3.145	Fr 05/22/20	100.071	3.141	Fr 05/01/20	100.899	3.038
Th 06/11/20	100.039	3.145	Th 05/21/20	100.156	3.130	Th 04/30/20	100.949	3.032
We 06/10/20	100.057	3.142	We 05/20/20	100.356	3.105	We 04/29/20	100.827	3.047
Tu 06/09/20	100.060	3.142	Tu 05/19/20	100.922	3.034	Tu 04/28/20	100.583	3.077
Mo 06/08/20	100.049	3.143	Mo 05/18/20	101.005	3.024	Mo 04/27/20	100.427	3.096

OAS1 function

OPTION-ADJUSTED SPREAD ANALYSIS

DOW CHEMICAL CO DOW3.15 08/15/29 100.042/100.047 (0.978/0.724) BVAL

Calculate Price OAS (bp) Volatility

(P,0,V) (P) 100.0470 (0) +60.18 (V) 95.17

Cusip / ID# 26054LJ79 Option Px Value: -16.98

Settle 6/30/2020 Bench settle 6/29/2020 Vega: 0.00

Spread 298.1bp vs GMO B 0 12/24/20 Govt @0.16 (0.162)

2) Customize

Curve CMT Semi

Const. Mty Tsy Cu

Dated 6/26/2020

Settle 6/30/2020

Shift +0 (bps)

Yield Spread

3m	0.135
6m	0.160
1y	0.155
2y	0.164
3y	0.182
4y	0.238
5y	0.296
7y	0.482
10y	0.642
20y	1.152
30y	1.376

{NUM}<G0> for:

3) Call Schedule

Yld	OAS Method	Option Free	To Call on 7/ 6/2020	To Mty
		1.177	0.326	3.144
		58.1	19.1	254.8
M Dur	0.51	0.02	7.79	
Risk	0.52	0.02	7.88	
Cnvx	-2.03	0.00	0.70	

Model L=Lognormal

Exercise Premium 0.00

88) REFRESH

B 0 12/24/20	↑ 0.1600	--	0.1650 / 0.1600	0.167 / 0.162
	At 13:42		-- x --	Source BGN
B 0 12/24/20 Govt		Settings	Actions	Page 1/11 Security Description: Bond
		95 Buy	96 Sell	
25 Bond Description	26 Issuer Description			
Pages	Issuer Information		Identifiers	
11 Bond Info	Name TREASURY BILL		ID Number	9127963L1
12 Addtl Info	Industry Treasury (BCLASS)		CUSTIP	9127963L1
13 Covenants	Security Information		ISIN	US9127963L18
14 Guarantors	Issue Date	06/25/2020	SEDOL 1	BL5B566
15 Bond Ratings	Interest Accrues		FIGI	BBG00VLFHK10
16 Identifiers	1st Coupon Date		Issuance & Trading	
17 Exchanges	Maturity Date	12/24/2020	Disc @ Issue	.175000
18 Inv Parties	Floater Formula	N.A.	Risk Factor	.487
19 Fees, Restrict	Workout Date		Amount Issued	56068 (MM)
20 Schedules	Security Type	USD	Amount Outstanding	56068 (MM)
21 Coupons	Type	ZERO	Minimum Piece	100
Quick Links	Calc Type	DISCOUNT	Minimum Increment	100
32 ALLQ Pricing	Day Count	ACT/360	SOMA Holdings	N.A.
33 QRD Quote Recap	Market Sector	US GOVT		
34 CACS Corp Action	Country	US		
35 CN Sec News	Currency	USD		
36 HDS Holders				
60 Send Bond	TENDERS ACCEPTED: \$54000MM.			

B 0 12/24/20	↑ 0.1600	--	0.1650 / 0.1600	0.167 / 0.162		
	At 13:42		-- x --	Source BGN		
9127963L Govt		96 Export	97 Settings	Page 1/1 Historical Price Table		
B 0 12/24/20			High	.1675 on 06/23/20		
Range	06/22/2020 - 06/26/2020	Period	Daily	Low	.1625 on 06/24/20	
Market	Mid Line	Currency	USD	Average	.1643	
View	Price Table	Source	BGN	Net Chg	-0.0016	
					-0.95%	
	Date	Mid Line	Mid YTM	Date	Mid Line	Mid YTM
Fr	06/26/20	.1625	.165			
Th	06/25/20	.1650	.167			
We	06/24/20 L	.1625	.165			
Tu	06/23/20 H	.1675	.170			
Mo	06/22/20	.1641	.166			

HOAS function

HISTORICAL OAS						
DOW 3.15 08/15/29						
	Date	Price	OAS	Volatility	Duration	Convexity
Thursday	6/25/20	100.04	80.31	95.	.59	-2.34
Wednesday	6/24/20	100.06	-12.70	101.	.03	-.10
Tuesday	6/23/20	100.07	88.29	93.	.75	-2.98
Monday	6/22/20	100.07	85.13	95.	.73	-2.89
Friday	6/19/20	100.04	83.67	97.	.71	-2.83
Thursday	6/18/20	100.04	83.49	97.	.72	-2.88
Wednesday	6/17/20	100.06	4.75	101.	.12	-.47
Tuesday	6/16/20	100.06	106.62	100.	1.22	-4.85
Monday	6/15/20	100.05	118.42	101.	1.49	-5.86
Friday	6/12/20	100.04	113.87	106.	1.36	-5.45
Thursday	6/11/20	100.04	109.85	106.	1.26	-5.02
Wednesday	6/10/20	100.05	107.59	98.	1.12	-4.45
Tuesday	6/ 9/20	100.06	95.35	98.	1.31	-5.22
Monday	6/ 8/20	100.05	105.86	89.	1.37	-5.46
Friday	6/ 5/20	100.04	102.02	90.	1.47	-5.88
Thursday	6/ 4/20	100.04	89.49	90.	.98	-3.91
Wednesday	6/ 3/20	100.07	-47.60	99.	.01	-.03
Tuesday	6/ 2/20	100.07	40.04	98.	.30	-1.17
Monday	6/ 1/20	100.06	81.46	100.	.63	-2.50
Friday	5/29/20	100.04	117.82	88.	.92	-3.66

25) Bond Description		20) Issuer Description	
Bond Info		Issuer Information	
Name: FREEPORT-MCMORAN INC		Industry: Metals and Mining (BCLASS)	
Country: US		Currency: USD	
Rank: Sr Unsecured		Series: Fixed	
Coupon: 5.250000		Type: Fixed	
Cpn Freq: S/A		Day Cnt: 30/360	
Maturity: 09/01/2029		Iss Price: 100.00000	
MAKE WHOLE @50.000000 until 09/01/24/ CALL ...		Interest Accrual Date: 08/01/2019	
Iss Sprd +335.00bp vs T 2 3/8 05/15/29		1st Settle Date: 08/15/2019	
Calc Type (1)STREET CONVENTION		1st Coupon Date: 03/01/2020	
Pricing Date: 08/01/2019		Par Amount: 1,000.00	
Interest Accrual Date: 08/15/2019		Book Runner: JOINT LEADS	
1st Settle Date: 08/15/2019		Reporting: TRACE	
1st Coupon Date: 03/01/2020			

25) Bond Description		20) Issuer Description	
Schedules		Call Schedule	
Call with minimum 30 days notice		Callable on and anytime after date(s) shown	
		Date	Price
		09/01/2024	102.625
		09/01/2025	101.750
		09/01/2026	100.875
		09/01/2027	100.000

OPTION-ADJUSTED SPREAD ANALYSIS					
FREIGHT-MCMORAN FCX5 3/4 09/01/29 102.401/103.012 (4.848/4.748) BVN4					
Calculate	Price	OAS (bp)	Volatility	2) Customize	
(P,0,V) (P)	103.0120	0) 412.6	V) 76.76	Curve	CMT
Cusip / ID#	35671DCD5	Option Px Value:	-1.14	Const. Mty Tsy Cu	
Settle	6/30/2020	Bench settle	6/30/2020	Dated	6/26/2020
Spread	435.4bp vs 7Y	T 0 1/2	06/30/27 Govt @100-3+	Settle	6/30/2020
				Shift	+0 (bps)
{NUM}<GO> for:	OAS	Option	To Call on	Yield Spread	
3) Call Schedule	Method	Free	9/ 1/2024	3m	0.135
Make whole	Yld	4.687	5.012	6m	0.160
provision	Sprd	408.6	476.3	1y	0.155
ignored	M Dur	6.31	3.66	2y	0.166
	Risk	6.61	3.83	3y	0.182
	Cnvx	-1.77	0.16	4y	0.239
				5y	0.297
				7y	0.484
				10y	0.644
				20y	1.151
				30y	1.374
	Model	L = Lognormal			
	Exercise Premium	0.00			
				88) REFRESH	

HISTORICAL OAS						
FCX 5 ¼ 09/01/29						
	Date	Price	OAS	Volatility	Duration	Page 1 / 11 Convexity
Thursday	6/25/20	102.40	420.89	73.	6.50	-1.11
Wednesday	6/24/20	103.00	411.38	73.	6.33	-1.59
Tuesday	6/23/20	103.36	401.37	68.	6.29	-1.64
Monday	6/22/20	104.01	389.62	70.	6.13	-1.98
Friday	6/19/20	104.08	389.55	70.	6.09	-2.12
Thursday	6/18/20	104.21	386.52	70.	6.07	-2.14
Wednesday	6/17/20	104.48	381.10	69.	6.01	-2.28
Tuesday	6/16/20	104.89	372.53	65.	5.93	-2.44
Monday	6/15/20	103.92	389.09	64.	6.19	-1.84
Friday	6/12/20	103.92	391.94	69.	6.17	-1.96
Thursday	6/11/20	103.13	407.36	71.	6.37	-1.51
Wednesday	6/10/20	105.64	360.53	65.	5.70	-3.02
Tuesday	6/ 9/20	105.61	350.64	58.	5.92	-2.06
Monday	6/ 8/20	106.73	327.63	56.	5.71	-2.28
Friday	6/ 5/20	106.21	332.90	55.	5.91	-1.76
Thursday	6/ 4/20	105.79	348.32	58.	5.88	-2.19
Wednesday	6/ 3/20	105.82	353.26	65.	5.77	-2.61
Tuesday	6/ 2/20	105.80	357.46	69.	5.71	-2.99
Monday	6/ 1/20	104.30	390.30	73.	6.01	-2.62
Friday	5/29/20	102.83	416.77	72.	6.43	-1.58

AS of 25 Jun									
AZ909662 Corp									
FCX 5 ¼ 09/01/29									
Range 08/02/2019 - 06/25/2020									
Market Mid Line Mid YTM									
View Price Table									
Source BVN4									
Page 1/6 Historical Price Table									
High 108.642 on 01/16/20									
Low 79.914 on 03/24/20									
Average 101.171 5.120									
Net Chg 4.300 4.37%									
Date	Mid Line	Mid YTM	Date	Mid Line	Mid YTM	Date	Mid Line	Mid YTM	Mid YTM
Fr 06/26/20			Fr 06/05/20	106.406	4.397	Fr 05/15/20	97.175	5.644	
Th 06/25/20	102.706	4.879	Th 06/04/20	106.001	4.449	Th 05/14/20	97.432	5.607	
We 06/24/20	103.202	4.813	We 06/03/20	105.973	4.453	We 05/13/20	98.214	5.497	
Tu 06/23/20	103.564	4.765	Tu 06/02/20	106.000	4.450	Tu 05/12/20	99.545	5.312	
Mo 06/22/20	104.227	4.678	Mo 06/01/20	104.500	4.645	Mo 05/11/20	99.045	5.381	
Fr 06/19/20	104.396	4.656	Fr 05/29/20	103.031	4.838	Fr 05/08/20	98.897	5.401	
Th 06/18/20	104.447	4.649	Th 05/28/20	103.598	4.764	Th 05/07/20	98.801	5.415	
We 06/17/20	104.834	4.599	We 05/27/20	102.106	4.962	We 05/06/20	98.798	5.415	
Tu 06/16/20	105.122	4.561	Tu 05/26/20	103.491	4.778	Tu 05/05/20	98.704	5.428	
Mo 06/15/20	104.124	4.692	Mo 05/25/20			Mo 05/04/20	97.540	5.591	
Fr 06/12/20	104.124	4.692	Fr 05/22/20	102.255	4.943	Fr 05/01/20	97.491	5.598	
Th 06/11/20	103.665	4.753	Th 05/21/20	101.508	5.043	Th 04/30/20	99.022	5.384	
We 06/10/20	105.895	4.462	We 05/20/20	101.500	5.044	We 04/29/20	98.460	5.462	
Tu 06/09/20	105.960	4.454	Tu 05/19/20	98.562	5.448	Tu 04/28/20	96.602	5.724	
Mo 06/08/20	106.952	4.327	Mo 05/18/20	98.477	5.460	Mo 04/27/20	95.279	5.914	

A.5 Implied volatility

Bloomberg calculates implied volatility for FX options and reports them as curves. For example, the implied volatility for the 6-month GBP/CAD currency pair is reported in the GBPCADV6M BGN Curncy Bloomberg curve. The Bloomberg print screen for the curve is illustrated below.

Exhibit A.7 Bloomberg screen for the 6-month GBP/CAD implied volatility

GBPCADV6M BGN Curncy		Option Volatility Description	
GBPCAD 6 Month ATM Implied Volatility			
Implied volatility is a measure of the market expected future volatility of a currency exchange rate from now until the maturity date. The future volatility is the single undeterminable variable in the common Black Scholes option pricing model. Bloomberg ATM implied volatilities can be used to obtain the correct Black Scholes price for a delta neutral straddle struck at maturity.			
British Pound GBP		Canadian Dollar CAD	
1) Economic Releases	3) Economic Releases	GBPCAD 6 Month Pricing Details - BGN	
2) Settlement Calendar	4) Settlement Calendar	Current	Open
		ATM	9.7375
		25D RR	-1.4325
		25D BF	-1.2975
		6M Fwd	.3650
			11.47
			Yesterday
			9.7300
			-1.3200
			.3725
			11.49
Security Ticker: GBPCADV6M FIGI: BBG0013QKC61 Option Type: At-The-Money Tenor: 6M Source: BGN - Bloomberg BGN Expiration Date: Mar 16, 2021 Days Until Expiration: 181 Trading Hours: 17:00 - 16:59 History Since: Mar 08, 2005		Option Volatility Analytics 5) ALLQ All Quotes 6) OVDV FX Volatility Surface 7) OVML FX Option Valuation 8) VOLC FX Volatility Comparison 9) WVOL World FX Volatility Matrix 10) OVRA FX Portfolio Risk 11) XOPT FX Option Central 12) FXTF Related Instruments	

Bloomberg allows to set a custom range of dates and custom period tick starting from 1 minute tick. An illustrative print screen of implied volatility graph for the GBP/CAD pair is illustrated below.

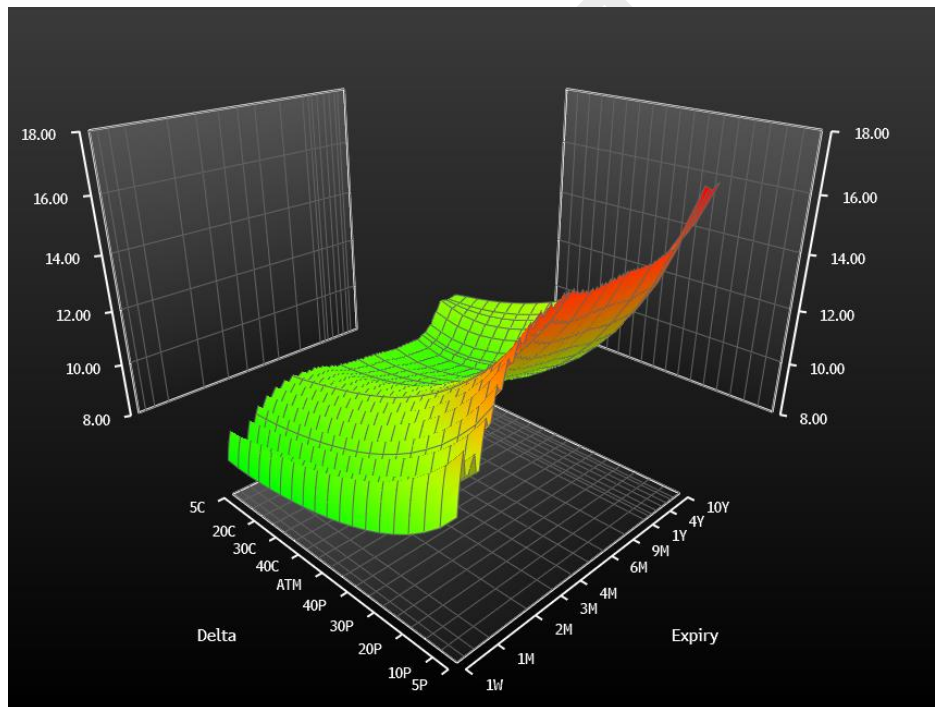
Exhibit A.8 Change in implied volatility over 1-day period using 1 minute tick



Alternatively, FX option implied volatility can be estimated using the following function: **FXIP> FX options and Volatility> (30) Volatility continuum – Real Vol ® to Future Implied Vol (I) | OVDV**. The function output is illustrated below.

GBPCAD Currency		90) Asset		91) Actions		92) Settings		Volatility Surface	
Bloomberg BGN		Offshore		New York 10:00		Weekdays		As of 09/16/2020	
Format		Side		Bid/Ask		Mid/Spread			
Exp	ATM Bid / Ask	25D Call GBP Bid / Ask	25D Put GBP Bid / Ask	10D Call GBP Bid / Ask	10D Put GBP Bid / Ask				
1D	6.620 / 14.520	5.115 / 15.615	5.471 / 15.959	0.676 / 20.634	1.219 / 21.291				
1W	6.560 / 9.760	5.932 / 9.988	6.556 / 10.634	4.035 / 11.828	5.141 / 12.896				
2W	7.235 / 9.835	6.656 / 9.929	7.446 / 10.749	5.251 / 11.404	6.632 / 12.793				
3W	7.750 / 9.925	7.200 / 9.925	8.117 / 10.878	5.989 / 11.051	7.666 / 12.764				
1M	8.115 / 10.015	7.558 / 9.932	8.639 / 11.051	6.434 / 10.816	8.469 / 12.901				
2M	8.690 / 10.590	8.074 / 10.436	9.344 / 11.766	6.976 / 11.324	9.337 / 13.783				
3M	8.990 / 10.475	8.416 / 10.254	9.766 / 11.664	7.591 / 10.949	10.066 / 13.534				
4M	8.905 / 10.320	8.366 / 10.114	9.719 / 11.531	7.634 / 10.821	10.086 / 13.399				
5M	8.789 / 10.177	8.273 / 9.983	9.643 / 11.423	7.616 / 10.733	10.108 / 13.364				
6M	8.740 / 10.120	8.240 / 9.937	9.635 / 11.408	7.613 / 10.707	10.103 / 13.347				
9M	8.675 / 9.975	8.160 / 9.750	9.706 / 11.384	7.537 / 10.433	10.319 / 13.391				
1Y	8.850 / 9.950	8.428 / 9.767	9.910 / 11.335	8.082 / 10.508	10.751 / 13.359				
18M	8.960 / 10.125	8.565 / 9.970	10.024 / 11.541	8.370 / 10.913	10.935 / 13.722				
2Y	8.980 / 10.280	8.603 / 10.157	10.059 / 11.761	8.559 / 11.371	11.077 / 14.213				
3Y	9.695 / 10.745	9.370 / 10.602	10.696 / 12.082	9.572 / 11.788	11.842 / 14.398				
4Y	10.140 / 11.240	9.828 / 11.097	11.117 / 12.578	10.075 / 12.350	12.285 / 14.990				

98) Legend		99) Quick Pricer		97) Option Pricing (OVML)		via USD					
Mty	Delta	Vol	Bid	Ask	Fwd	Mid	Deposit				
Exp	10/15/2020	Strike	49.981	1.7074	GBP Price	0.929%	1.142%	Spot	1.7078	CAD	0.141%



A.6 SDRV tool

The SDR Derivative volumes (SDRV) tool shows the individual FX options, swaps, CDS and other derivative contracts traded in the OTC markets. The print screen for the GBP / CAD vanilla options is illustrated in the print screen below.

Exhibit A.9 A sample of individual FX option contracts

Source		DTCC		Period		Custom		09/10/20		09/16/20	
1) Credit 2) Rates 3) FX 4) Comdty 5) Equity											
1) Options 2) NDF											
Underlying	Type	Strike	Expiry	Notional	Curr	Premium	Curr	Style	Code	Clr	Trd Time
GBP CAD											
1. GBP CAD Vanilla Option	C	1.78	12/18/20	5	GBP	22,550.56	GBP	EU	TR	U	09/16 12:1
2. GBP CAD Vanilla Option	P	1.63	12/18/20	5	GBP	29,475.00	GBP	EU	TR	U	09/16 12:1
3. GBP CAD Vanilla Option	P	1.78	12/18/20	5	GBP	28,951.69	GBP	EU	TR	U	09/16 12:0
4. GBP CAD Vanilla Option	P	1.63	12/18/20	5	GBP	32,265.00	GBP	EU	TR	U	09/16 12:0
5. GBP CAD Vanilla Option	C	1.7455	02/10/21	30	GBP	384,004.59	GBP	EU	TR	U	09/15 12:3
6. GBP CAD Vanilla Option	C	1.817	02/10/21	61	GBP	240,946.61	GBP	EU	TR	U	09/15 12:3
7. GBP CAD Vanilla Option	P	1.68	10/23/20	20	GBP	184,800.00	GBP	EU	TR	U	09/15 11:5
8. GBP CAD Vanilla Option	P	1.81	03/15/21	2	GBP	15,679.56	GBP	EU	TR	U	09/15 04:5
9. GBP CAD Vanilla Option	P	1.55	03/15/21	2	GBP	12,950.00	GBP	EU	TR	U	09/15 04:5
10. GBP CAD Vanilla Option	P	1.81	03/15/21	7	GBP	39,359.12	GBP	EU	TR	U	09/15 04:3
11. GBP CAD Vanilla Option	P	1.55	03/15/21	7	GBP	35,210.00	GBP	EU	TR	U	09/15 04:3
12. GBP CAD Vanilla Option	P	1.653	03/08/21	20	GBP	374,000.00	GBP	EU	TR	U	09/11 09:0
13. GBP CAD Vanilla Option	C	1.74	12/11/20	2	GBP	13,965.52	GBP	EU	TR	U	09/11 06:2
14. GBP CAD Vanilla Option	P	1.61	12/11/20	2	GBP	15,110.00	GBP	EU	TR	U	09/11 06:2
15. GBP CAD Vanilla Option	C	1.74	12/11/20	7	GBP	50,758.62	GBP	EU	TR	U	09/11 05:3
16. GBP CAD Vanilla Option	P	1.61	12/11/20	7	GBP	48,860.00	GBP	EU	TR	U	09/11 05:3
17. GBP CAD Vanilla Option	C	1.79	03/11/21	5	GBP	44,798.88	GBP	EU	TR	U	09/10 05:5
18. GBP CAD Vanilla Option	P	1.63	03/11/21	5	GBP	48,400.00	GBP	EU	TR	U	09/10 05:5

The tool reports the following information: type of the option (put / call); underlying FX notional amount, option strike price, option expiry date, option premium, option style (European / American), option exercise time. The tool shows instruments for one-month period only. The transactions data can be exported into an Excel file.

Appendix B Implementation of Lease Residual Value Risk Model

The lease model was presented in Section 2 and the risk-neutral probabilities and lease derivative prices were derived in Appendices B and C. In this section, the implementation of the lease valuation model is discussed in more detail. The discussion covers all steps starting from the data required for the model estimation (and respective data request to the client) and ending with the results presentation that are provided to the client.

B.1 Modelling background

The buyer of the residual value risk reimbursement contract owns a portfolio of assets which are leased to third-party lessees. The buyer projects a residual value of the asset at the end of each lease contract. The residual value determines the monthly fee charged in the lease contract. The buyer is exposed to the residual value risk in the event if the actual residual value is below the projected expected value.

The risk exposure is hedged by entering into the residual value risk reimbursement contract with the contract seller. The hedging contract specifies the strike price (e.g. 90% of the expected residual value) such that any decrease of the residual value below the strike price is compensated by the contract seller. The premiums are paid by the contract buyer on an annual basis. The contract may be rolled over (including the update of the contract premiums) on an annual basis.

B.2 Data

Requesting data is the first step required before the analysis can be started. The objective of the step is to ensure that the required data is available and is reviewed for missing or incorrect information.

B.2.1 Data request

The following lease data is requested for the valuation analysis:

1. Lease insurance **contract agreement**. The terms of the agreement include
 - ▶ Definitions of the projected and actual residual value of the cars
 - ▶ Schedule with the projected residual values
 - ▶ Reference to the methodology for the actual residual value estimation⁷
 - ▶ Strike price for the lease insurance claims
2. Individual **contract data**. The data includes for each specific car lease contract the following information
 - ▶ Car type, build year, purchase year, and purchase price
 - ▶ Lease issue date, termination date, monthly rate, and lease factor
 - ▶ Car estimated depreciation schedule and residual value
3. Historical data on lease insurance contract **claims**

⁷ The methodology should be requested from the client.

- ▶ For destroyed cars: the percentage of the destroyed cars in the overall portfolio and salvage values of the destroyed cars
 - ▶ For terminated leases, the residual car value as of the lease termination date
4. **Market data.** The following additional data is necessary or recommended for the valuation model
- ▶ Market risk-free rates
 - ▶ Market price data for the leased cars. Market data for individual car types are generally not publicly available but may be requested from the client who may maintain a database with the price information obtained through catalogues or other sources.

B.2.2 Lease insurance contract

The contracts are referred to as “Residual Value Risk Reimbursement” contracts.

Background

The contract background summarizes the following key facts: (i) the lease payments depend on the projected residual value of the underlying car assets; (ii) the lessor is exposed to the residual value risk: in the event that the actual market value of the car at the lease termination date is below the projected residual value, the lease payments do not fully compensate for the loss in the car value. As a result, the lessor incurs losses in the lease contract.⁸ The purpose of the lease insurance contract is to protect the lessor against the residual value risk exposure.

The contracts typically include the following definitions and terms.

Definitions

- ▶ Seller and Buyer full legal name
- ▶ Trade / effective date: the lease insurance issue / amendment date
- ▶ Projected Residual Value (V^R) is specified in the agreement schedule
- ▶ Strike Price (K) is calculated as a given percentage (s) of Projected Residual Value: $K = s \times V^R$
- ▶ Actual Residual Value (V^A) is assigned to the car at the end of the lease and is calculated as follows:⁹
 - ▶ The NPV of the lease cash flows over the car remaining useful life is estimated by the lease insurance Buyer
 - ▶ If the Seller and the Buyer disagree on the estimated actual residual value, the parties get the appraisal from a third-party to which they both mutually agree
 - ▶ If the parties disagree on the appraisal, they follow the dispute resolution process.

⁸ Note that the risk exposure is two-sided: the market value of the car on the lease termination date may be above the projected residual value. In this case, the lessor obtains an additional gain from the car residual value.

⁹ Note that the definition of the Actual Residual Value assumes that the car is not traded in the market and the direct market value of the car is not available.

Terms

- ▶ Reimbursement Payout value is calculated as the difference between strike price (e.g. 90% of V^R) times the Projected Residual Value and Actual Residual value or zero (whichever is greater):

$$P = \max[0, s \times V^R - V^A]$$

- ▶ Request for Risk Reimbursement Payout is provided within a specific period and shall be supported by the documentation with the Actual Residual Value appraisal
- ▶ Premiums and Payment Terms. The Premiums (F) paid by Buyer to Seller are specified in the agreement schedule. Premiums are calculated as a percentage (f) of the Projected Residual Value and are allocated annually (on XXX date of each year) on a straight-line basis (consistently with the Projected Residual Value depreciation schedule)

$$F = f \times V^R$$

- ▶ Car Additions. New cars are added automatically to the portfolio covered automatically by the insurance contract. The applicable premiums are calculated as per the agreement premium schedule. Upon the addition of new cars, the Seller has a specific period to object to the new cars addition to the portfolio covered by the insurance contract.
- ▶ Lease Termination Terms. The lease insurance agreement is terminated in the even if (i) one of the parties becomes insolvent; (ii) one of the parties undergoes xx% change in ownership; or (iii) the Buyer disposes from xx% of the portfolio assets within a calendar year.¹⁰

B.2.3 Data review and transformation

The provided data on individual lease contracts typically needs to be reviewed and transformed to be applied in the valuation model. The following steps are typically performed to present the data in the appropriate format.

1. Data reduction. Typically, the data set includes a significant amount of redundant information which is not relevant for the valuation purposes. The data set is reviewed to ensure that all necessary fields are present and then the data set is reduced to include only necessary fields. The fields are listed in Appendix E.1.1.
2. Data aggregation. The data set often includes multiple records of effectively identical leases. An identification label is created for each specific contract type and the leases are grouped based on the identification label. The id can be built, for example, as a combination of the following fields:
3. lease id = (car type, lessee id, car build date, car purchase date, lease termination date, lease monthly fee)
4. Data screening. Some lease contracts may have missing fields (which are required for lease valuation) or the field values result in inconsistent pricing.¹¹ The contracts should be screened to remove the outliers from the valuation model. The contracts should be reviewed and discussed with the client separately to identify the source of values inconsistency.

¹⁰ Formally, the agreement should specify the payouts in the event of assets disposal and termination of the respective insurance contracts. In the absence of such terms, it is either assumed that the payouts are zero or the payouts are estimated based on the residual value of the insurance contracts.

¹¹ The data should be screened for (i) missing fields values; (ii) zero (or small) car purchase price values; (iii) zero (or small) lease monthly fee values.

After performing the steps above, a data set is created which is used to estimate the parameters of the residual risk valuation model.

B.3 Modelling approaches

Based on the analysis of the insurance claims, the selected modelling approach can be broadly classified either as a (i) CDS approach or (ii) as a Black-Scholes approach.

1. **CDS approach** should be selected whenever the insurance claims are consistent with the evidence that the primary risk which triggers the insurance claims is the destruction of the lease assets. Under the approach, the value of the asset is modelled as a binary process with the states corresponding to destroyed / non-destroyed asset.
2. **Black-Scholes approach** should be applied whenever the insurance claims are consistent with the evidence that the risk which triggers the insurance claims is related to the continuous depreciation of the lease asset value. The claim is submitted in the event when the actual depreciation is accelerated relative to the expected depreciation resulting in the lease value being below the contract strike price at the lease contract maturity. The continuous stochastic depreciation of the lease asset can be attributable to various factors such as technological risk, market risk, regulatory risk, and other.

Under both the CDS and Black-Scholes approaches, the residual risk reimbursement contract can be priced either as an insurance contract or a derivative contract. The distinction between the two approaches can be summarized as follows.

1. Under the **insurance approach**, the price is derived based on the estimation of the costs and profit margins of the risk reimbursement contract. The costs can be broken down into (i) the costs related to the insurance claims payments (when the insurance event is triggered); and (ii) administrative costs related to issuing, valuation, execution, and monitoring the contracts. The costs and profit margins are estimated based on the analysis of historical financial statements and insurance claims of the tested entities or comparable entities with publicly available market data.
2. Under the **derivative approach**, the price of the residual value risk reimbursement contract is estimated as a derivative of market prices including (i) lease fees and (ii) applicable market risk-free rate. The derivative approach assumes that implied costs and profit margins are incorporated into the lease fees and are respectively derived from the lease fees. More precisely, the residual value risk reimbursement contract is replicated using a portfolio of the lease contract and a risk-free bond and the price is derived based on the replication portfolio.

Implementation of the approach is discussed in detail in the sections below.

B.4 Estimation of model parameters

The objective of the parameter estimation step is to (i) estimate the expected depreciation term structure Λ_t ; (ii) estimate the depreciation risk volatility parameter σ ; and, if applicable, (iii) to estimate the asset destruction hazard rate $\gamma(s)$.

B.4.1 Approach

Suppose that the following lease portfolio data is available for analysis:

1. The build date and asset (product) type¹² for each lease underlying asset;
2. Each lease contract purchase date and purchase price. Lease purchase price represents the underlying asset residual value effective as of the lease purchase date;
3. Residual value estimation method applied by the lessor;
4. Each lease contract monthly payment and lease factor;
5. Tax rate applicable to the lease contracts.

The parameters can be used to specify the id of each lease contract in the following format:

id = product type – lessee id – build date – purchase date – lease termination date – monthly payment – lease factor.¹³

The **empirical** expected depreciation term structure Λ_t is estimated as follows.¹⁴ The function Λ_t is estimated based on the lease contract purchase date and purchase price information. The purchase date is converted to the asset age value t . The purchase price corresponds to the residual value S_t . The data is constructed for each individual lease and is grouped by the underlying asset type i . The function Λ_t is estimated by performing a fixed-effect panel regression analysis for the $S_{i,t}$, where $S_{i,t}$ is the market residual value of the asset type i with age t . The model specification is based on the following two assumptions:¹⁵

1. The term structure does not change over time. The model specification uses only the asset age as parameter but not the date when asset was built.
2. The term structure does not change with the asset type. The parameters Λ_t are assumed to be independent of asset type indexed by i .

Based on the estimated lease asset depreciation term structure, implied lease factor is estimated as follows:

$$f = \frac{\frac{F \times (s - t)}{1 + \tau} - (S_t - S_s)}{(S_t + S_s) \times (s - t)}$$

Implementation of the approach is illustrated in Appendix E.

B.4.2 Implementation

The valuation formulas presented in Appendices B.4.4 and C.2.4 require the following parameters to be estimated:

1. Risk-free rates

¹² In the case of equipment leasing, the asset type can correspond to different equipment products. For example, in the case of railcar leasing the product types can be different models of tank cars, covered hopper railcars, etc.

¹³ The id assumes that all contracts in the portfolio are renewed on the same date (and therefore have the same lease issue date).

¹⁴ An alternative approach is to estimate the **implied** depreciation term structure based on the implied residual value used by the lessor to calculate the lease monthly rate. The lease monthly rate, lease factor, and tax rate are used to estimate the implied residual value applied by the lessor in the monthly rate calculations. The implied depreciation term structure is calculated then based on the implied residual value sample using a fixed-effect panel regression analysis as discussed above. The monthly rate is calculated using the following equation: $F = \left[\frac{S_t - S_s}{s - t} + f \times (S_t + S_s) \right] \times (1 + \tau)$. The implied residual value is estimated from the equation as follows: $V_s = \frac{S_t \times (f \times (s - t) + 1) - \frac{F \times (s - t)}{1 + \tau}}{1 - f \times (s - t)}$.

¹⁵ Validity of the assumptions is discussed in Katharina Nau, "An Empirical Analysis of Residual Value Risk in Automotive Lease Contracts".

2. Market-based car residual value schedule

Estimation of the risk-free rates is discussed in a separate guide ([reference]). This section presents details of the estimation procedure for the car residual value schedule.

The residual value schedule is estimated based on the following functional form assumption:

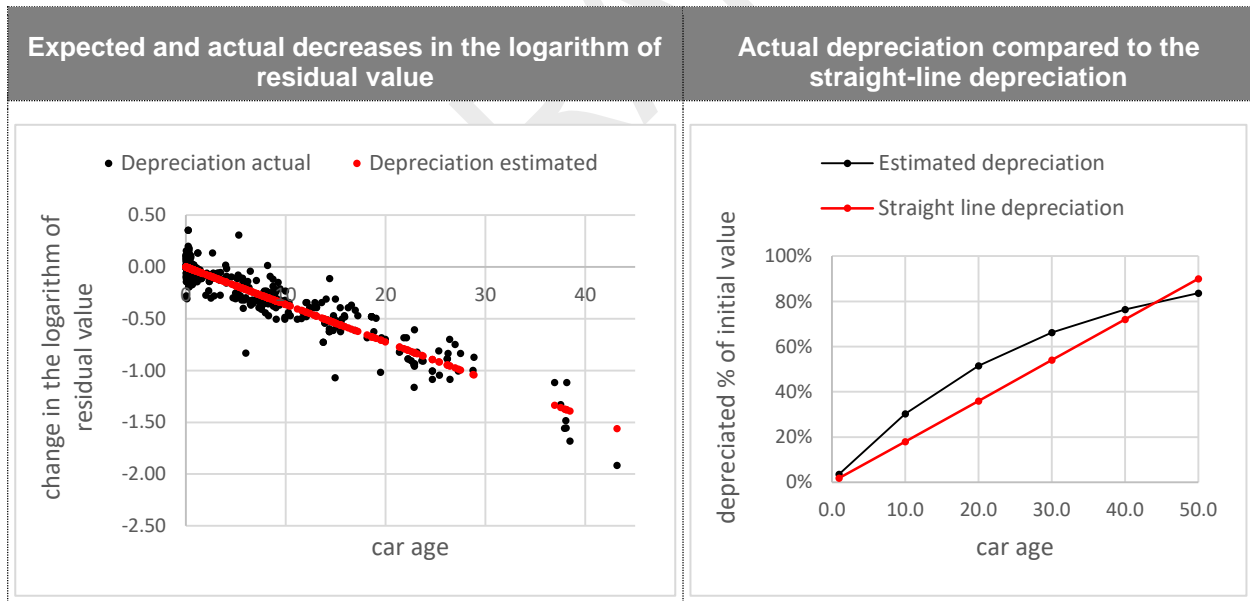
$$(B.1) \quad s_{i,t} = s_{i,0} - \Lambda_t + \sigma W_t$$

where function Λ_t is assumed to have a linear form

$$(B.2) \quad \Lambda_t = \lambda t$$

and $s_{i,t}$ is the natural logarithm of the type i car residual value at age t (and $s_{i,0}$ is the car value at the car build date). The unknown parameters $v_{i,0}$ and λ are unknown parameters estimated using a fixed-effect panel regression model.¹⁶ Results of the regression model estimation are illustrated in the diagram below, where the red line represents the linear downward trend in the logarithm of the residual value ($-\lambda t$) and the black dots represent the sample of actual decreases in the logarithm of the residual value ($(-\lambda t + \sigma W_t)$). Exhibit below also compares the actual depreciation (estimated based on the data) against the straight-line depreciation (assuming 50-year term and 10% residual value after 50years).

Exhibit B.1 Residual value depreciation schedule



Under the Black-Scholes model specification, the residuals σW_t in the panel data model specification are serially correlated and heteroscedastic ($var \sigma W_t = \sigma^2 t$). The exhibit above does not show the increase in the residuals standard error with the increase in the car age. Therefore, the Black-Scholes model needs to be modified to be consistent with the empirical data.

¹⁶ Depending on the data patterns, more complex functional form for the function Λ_t can be selected and tested.

B.5 Residual risk valuation for individual contracts

This section presents the model in which a residual value risk reimbursement contract is issued for each individual lease asset. The portfolio of the residual value risk reimbursement contracts is priced as a sum of individual contract prices.¹⁷

B.5.1 CDS approach

The CDS approach should be selected whenever the data indicates that majority of residual value risk claims are related to the destruction of underlying lease assets value.

The valuation can be performed using two alternative approaches: insurance approach and derivative approach.

CDS / Insurance approach

Under the insurance approach, the fee is estimated as the sum of the following components:

$$\text{Fee} = \text{Expected Loss} + \text{Expense Load} + \text{Profit Provision}$$

Each component of the fee is estimated as follows:

1. **Expected Loss.** Expected losses are calculated either (i) based on the client's historical losses and estimation of the respective empirical loss distribution; or, alternatively, (ii) based on the publicly available market historical loss data for comparable lease assets. The empirical loss distribution is estimated on an annual basis and losses are estimated as a percentage of the asset residual value.
2. **Expense Load.** The Expense Load encompasses the costs of doing business incurred by the insurer as a result offering such coverage. The cost includes the following components: production expenses (e.g., marketing expenses, broker/agent fees); general expenses (e.g., personnel, systems costs); other administrative expenses (i.e., miscellaneous expenses not covered elsewhere); and taxes, licenses and fees. The Expense Load can be estimated either based on the historical financial statements of the insurer / guarantor (if available) or based on the market financial data of the companies which perform similar functions.
3. **Profit Provision.** The profit margin is estimated either (i) based on the value-at-risk (VaR) analysis as a compensation for the insurer's capital balances required to guarantee that the insurance claims will be paid when due; or (ii) based on the transfer pricing analysis of profit margin estimated based on the market profit margin data estimated using a sample of comparable companies.

Structuring and valuation of the contract as an insurance contract should take into consideration that the seller of the contract may be required to meet the regulation requirements of an insurance company (for example, it may be required to have adequate capitalization).

CDS / Derivative approach

Under the CDS approach, the lease residual value risk reimbursement contracts are modelled similar to the amortized bond CDS contracts. Derivative approach is based on arbitrage-free prices which incorporates both the expected cost and profit components into the derivative price. Similar to the insurance approach, the derivative approach estimates the value of the reimbursement contract based on the hazard

¹⁷ In the following section, the pricing approach is presented for the case when a single residual value risk reimbursement contract is issued for the whole portfolio of lease assets.

rate of the asset destruction. Unlike the insurance approach, the implied hazard rate is derived from the lease market fees and not from historically observed asset destruction hazard rates.

The steps of the valuation approach are summarized as follows:

1. **Valuation date.** The market lease fee is observed only at the lease **issue date**. Therefore, the price information is available only as of the lease issue date.
2. **Lease fee adjustment.** The lease periodic monthly fee is adjusted for the operational cost to produce equivalent amortized bond periodic payments. The cost-adjusted lease fee is interpreted as the sum of (i) lease asset amortized value and (ii) interest payment on the leased asset;

$$D_0 = F \times m^{op}$$

3. **Lease asset amortization (λ).** Amortization schedule is estimated based on the historical panel regression data of the underlying asset residual values. Annual amortization is estimated as a percentage of the asset residual value:

$$(B.3) \quad v_t = v_0 - \lambda t + \sigma(t) \times W_t$$

where $v_t = \ln V_t$ and V_t is the residual value of the asset with age t .

Alternatively, amortization schedule is estimated assuming straight-line depreciation of the asset. The straight-line depreciation assumes a given (50-year) useful life of the asset (starting from the date when the asset was built) and a given residual value of the asset at the end of the useful life (e.g. 10% of the asset initial value). The asset initial value is estimated as the purchase price at the date when the asset was purchased.¹⁸

4. **Implied lease interest rate (i).** The cost-adjusted lease fee is converted into the implied interest rate by decomposing the fee into the amortized value and interest components:

$$(B.4) \quad S_0 = S_0 \times e^{-dT} + D_0 \times \frac{1 - e^{-i \times \frac{n+1}{12}}}{1 - e^{-\frac{i}{12}}}$$

where d is implied dividend rate; $i = d - \lambda$ is the implied lease interest rate; and n is the number of fixed monthly dividend payment over the term of the lease.

5. **Implied premium (f).** The implied premium is estimated as a difference between implied interest rate and risk-free rate (or group refinancing rate in the case of the intercompany lease derivatives. The group refinancing rate is used to adjust for the counter-party risk).

$$(B.5) \quad f = (i - r) \times \frac{k - \alpha}{1 - \alpha}$$

where parameter r represents a one-year risk-free (or group refinancing) rate, k is the strike price parameter (represented as a percentage of the residual value); and α is the recovery rate parameter.

6. **Contract roll-over.** The derivative contracts are rolled-over and re-priced on an annual basis. The destroyed lease assets are removed from the portfolio and the insurance claims are settled. The

¹⁸ Note that in many cases the asset purchase date is different from the asset's built date and the asset purchase price already includes asset depreciation that accumulated from the asset built date.

fee f is applied to all remaining contracts in the portfolio.¹⁹ The effective maturity of each derivative contract is assumed to be **one year**. Therefore, a one-year risk-free (group refinancing) rate is applied in the premium estimation

The model parameters can be grouped as follows:

1. Parameters F (lease monthly fee), k (strike price), and T (lease term) are specified by each lease contract or residual risk reimbursement contract;
2. Parameter m^{op} is estimated from the lessor historical financial statements;
3. Parameter λ is estimated based on the panel regression model for the lease asset residual value data;
4. Parameter r is estimated as follows:
 - ▶ If r represents risk-free rate, then it is estimated based on the respective risk-free indices;
 - ▶ If r represents group refinancing rate, then it is estimated based on the interest benchmarking analysis performed for the group (or risk reimbursement contract seller).
5. Parameter α is estimated based on the observed historical recovery rates of the destroyed assets. By default, $\alpha = 0$;
6. Parameter i is estimated as implied parameter in equation (B.4);

Illustrative results of the approach implementation for an actual portfolio of railcar leases are provided below.

[results]

B.5.2 Black-Scholes approach

The Black-Scholes approach should be selected whenever the data indicates that majority of residual value risk claims are related to the decrease of continuously changing residual value below the contract strike price.

Similar to the CDS approach, the valuation can be performed using two alternative approaches: insurance approach and derivative approach.

Insurance approach

The insurance approach under the Black-Scholes valuation methodology is similar to the CDS / insurance approach. The total fee is presented as a sum of three components described above. The estimation methodology for the Expense Load and Profit Provision components is similar to the methodology described for the CDS / insurance approach.

The expected cost component t is estimated based on the residual value regression model described by the equation

$$s_{i,t} = s_{i,0} - \Lambda_t + \sigma W_t$$

¹⁹ If the estimated fee depends statistically on the residual value (age) of the lease asset or lease maturity term, then the fee is adjusted accordingly for all contracts remaining in the portfolio.

The expected cost for the lease asset i is calculated formally as follows

$$l = E \left[(\ln K - v_{i,r})^+ \right]$$

where K is the contract strike price. The expected loss is normalized then as follows:

$$f^L = \frac{e^l}{S_0}$$

Derivative approach

Under the BS / derivative approach, steps one to three are the same as under the CDS / derivative approach. The difference starts from step four.

1. **Implied residual value drift parameter.** The implied drift parameter is estimated as follows

$$(B.6) \quad \mu \rightarrow - \left(\lambda + (i - r) + \frac{\sigma^2(T)}{2T} \right)$$

where $d = \lambda + i$ is implied dividend parameter, which is estimated from the following equation:

$$(B.7) \quad d = \frac{D_0}{T \times S_0} \times \frac{1 - e^{-r \times (n+1)/12}}{1 - e^{-r/12}}$$

where T is the lease maturity term and $n \times D_0$ is the total lease fee paid during the term of the lease contract. Note that the drift parameter is lower than the parameter λ in the BS / insurance approach (assuming that $i - r \geq 0$) and, therefore, it can be interpreted as that the parameter includes both the cost and profit components;

2. **Volatility parameter.** The implied volatility function $\sigma^2(T)$ is estimated from the linear regression model (c.4.1);
3. **Put option price.** The value of the put option is estimated as

$$(B.8) \quad V^{put} = -N(-d_1) \times S e^{-dT} + N(-d_2) \times K e^{-rT}$$

Where parameters d_1 and d_2 are estimated as follows

$$(B.9) \quad \begin{cases} d_1 = \frac{1}{\sigma(T)} \times \left[\ln \frac{S}{K} + (r - d)T + \frac{\sigma^2(T)}{2} \right] \\ d_2 = d_1 - \sigma(T) \end{cases}$$

4. **Contract roll-over.** In the case of Black-Scholes model, the strike price is typically set as of the lease termination date and, therefore, rolling and re-pricing the reimbursement contracts on an annual basis is inconsistent with setting the strike price at the contract maturity. Re-pricing of the residual risk reimbursement contract requires also the annual re-evaluation of the underlying lease asset.

The model parameters can be grouped as follows. Parameters in groups (i) – (iv) are the same as in the CDS / derivative approach. Other parameters are grouped as follows:

1. Parameter d is estimated based on the equation (B.7);
2. Volatility function $\sigma^2(T)$ is estimated based on the panel regression model (together with parameter λ);
3. Parameter S_t is provided by the client. Alternatively, parameter S_t is calculated using (i) straight-line depreciation $S_t = S_0 - \delta t$ (where δ is depreciation parameter²⁰); or (ii) expected market depreciation $S_t = S_0 e^{-\lambda t}$.

Illustrative results of the approach implementation for an actual portfolio of railcar leases are provided below.

[results]

B.5.3 Mixed CDS / Black-Scholes approach

B.5.4 Summary of the residual risk valuation modelling

A high-level summary of the valuation approach can be described as follows.

Selection of the approach

1. There are two alternative approaches to the residual risk hedging valuation: insurance approach and derivative approach;
2. The derivative approach estimates the price of the risk reimbursement contract as a derivative of the lease market fee (lease market price).
3. The derivative approach can be implemented either as a (i) CDS model or as a (ii) Black-Scholes model;

CDS approach

1. CDS approach assumes that the insurance event is triggered whenever the underlying asset is destroyed. The lease CDS model can be viewed as similar to the amortized bond CDS model. The equivalence of the two models can be presented as follows:
 - ▶ The destruction of the underlying asset is interpreted as a default on the bond. Therefore, the residual value risk is interpreted as the default risk when the underlying asset is destroyed;
 - ▶ The depreciation of the lease asset corresponds to the amortization of the bond;
 - ▶ Periodic lease fee includes two components: interest payment and the asset amortization payment.

²⁰ In general case, δ is calculated as $\delta = \frac{S_0 \times (1 - \rho)}{T_{life}}$, where S_0 is the price of the asset at the date when it was acquired, T_{life} is expected remaining life of the asset (e.g. 50 years after the asset was built); and ρ is the residual value of the asset after the end of the expected life (e.g. 10%).

2. There are certain differences between the lease CDS and amortized bond CDS models, which result in models' implementation differences:
 - ▶ Lease management includes certain operational costs. Therefore, the lease fee includes not only the profit margin which compensates for the time value of money and lessee default / asset destruction risk but must also compensate for the operating costs (equivalent of admin fee in bonds);
 - ▶ The lease depreciation schedule is random unlike the bond amortization schedule. There are different approaches to model lease depreciation schedule (e.g. assume straight-line depreciation or estimate the depreciation based on market data);
 - ▶ The terms of an amortized bond transaction include both the bond amortization schedule and bond interest rate. The terms of the lease agreement include the periodic lease fee which must be decomposed into the lease depreciation and interest components. Decomposition of the lease fee into two components is part of the analysis and the results of the analysis depends on the lease depreciation modelling approach.
3. Under the CDS approach, the periodic premium is calculated using the following equation

$$f = k \times (i - r)$$

where k is the strike price quoted as percentage of the asset residual value, i is the implied interest rate (estimated from the lease fee decomposition), and r is either the risk-free rate or the rate applicable to the 'insured' asset. In the latter case, the rate r models the counter-party default risk (which includes both the lessee default risk and the risk reimbursement contract seller default risk). In the case of the amortized bond transaction, i represents the bond yield rate.²¹

Black-Scholes approach

1. Black-Scholes approach assumes that the insurance event is triggered whenever the underlying asset is depreciated at accelerated rate than expected. The approach assumes continuous changes in the asset residual value (and, therefore, does not account for the asset destruction probability);
2. The Black-Scholes application to the lease asset residual value model can be implemented as a standard Black-Scholes model with implied dividend rate estimated based on the lease fee payments;
3. The asset depreciation empirical data is consistent with the geometric rate of reduction in the asset value and, therefore, consistent with the geometric Brownian motion.²² The volatility of the residual value does not show the linear increase $\sigma(T) = \sigma^2 \times T$ over time. Therefore, the $\sigma^2 \times T$ term in the Black-Scholes equation should be replaced with a generic $\sigma(T)$ value estimated from the residual value regression model;
4. The risk-neutral expectation of the residual value percentage change in the Black-Scholes model can be presented as follows:

$$\mu \Rightarrow -\left(\lambda + (i - r) + \frac{\sigma^2(T)}{2T}\right)$$

²¹ In a standard CDS model with no counter-party risk, r represents risk-free rate. In the model with the counter-party risk, r represents the interest rate applicable to the guarantor.

²² The geometric Brownian process assumption for the asset value is one of the key assumptions in the Black-Scholes model.

where λ is expected asset depreciation rate and i is implied interest rate. The risk-neutral parameter μ depends only on the implied dividend rate $d = \lambda + i$ and does not depend on a specific decomposition of the dividend rate into the asset depreciation rate and interest rate.

5. The put option value under the Black-Scholes approach is described by equations (B.8) and (B.9).
6. To apply the Black-Scholes equations, the market residual value S_t must be estimated as of the residual risk reimbursement contract valuation date (by default, the S_t can be estimated using straight-line or depreciation schedule or the schedule estimated from the regression model). For consistency with the Black-Scholes modelling approach, the maturity term of the contract should be set equal to the lease termination date²³.

B.6 Residual risk valuation for a portfolio of contracts

DRAFT

²³ The contract not rolled over and re-priced on the annual basis.

Appendix C `ac.finance.*` option pricing tools

C.1 `ac.finance.BSM`

C.2 `ac.finance.APM`

The objective of the `ac.APM` tool is to implement the models presented in this guide. Numerical implementation of the models allows to select flexibly modelling parameters and produce model specifications which do not have simple analytical solutions.

C.3 Overview

The models simulated by the `ac.APM` tool are similar to the interest rate models (implemented by the `ac.SRM` tool). The tool models stochastic processes with a one-dimensional space of process states. The model is a specific case of the one-dimensional controlled Markov process.

There are two key differences of the `ac.APM` tool from the `ac.SRM` tool and general Markov processes.

1. The `ac.APM` models both (i) the value of the underlying asset and (ii) the value of the derivative instrument. The tool allows to assess the impact of the changes in the parameters of the underlying asset (e.g. change in the dividend payout ratio) on the value of the derivative contract;
2. The `ac.APM` tool estimates risk-neutral probabilities as part of the derivative contract modelling. Estimation of the risk-neutral probabilities requires that the market completeness requirement is satisfied (number of modelled assets equals to the number of states).

To summarize, the `ac.APM` tool is implemented using the following principles:

1. The tool assumes market completeness (by default, two assets and two states in each period of time);
2. The tool assumes existence of the risk-free asset with continuous rate of return $r \times dt$;
3. The tool derives both the underlying asset price and derivative prices;
 - ▶ The dividend process is specified by the user and the stock price is derived from the dividend process;
 - ▶ The stock price is modelled directly as a function of an abstract stochastic factor. The process for the abstract stochastic factor is specified by the user;
4. If the asset price is derived from the dividend process, the price is calculated using the contraction mapping principle summarized as follows:
 - ▶ fix terminal price V_T ;
 - ▶ run backward recursion to estimate prices V_0 at $t = 0$;
 - ▶ At iteration n , set $V_T^{(n)} = V_0^{(n-1)}$ and re-run the backward recursion until $V_0^{(n)} = V_T^{(n)}$
5. For given estimated price process, estimate risk-neutral probabilities;
6. Estimate derivative prices

C.4 Model inputs

Model inputs include: (i) set of states for a one-dimensional stochastic factor; (ii) transition probabilities for the stochastic factor; (iii) collection of assets and mapping from the stochastic factor state into respective asset price; (iv) dividend function for each asset.

By default, one of the assets in the modelled collection of assets is a risk-free bond which maps each state of the stochastic factor into the bond price as follows: $B_{t+1} = (1 + r) \times B_t$, where r is the risk-free rate. The risk-free bond is modelled automatically by the tool and is not included in the collection of assets specified by the user.

Each process is modelled as a discrete tree (including continuous diffusion processes). In the tree model specification, the number of assets must be equal to the number of states to ensure that market completeness condition is satisfied, and risk-neutral probabilities can be derived from the model specification.²⁴ Note that risk-neutral probabilities are derived within the tool and not specified by the user.

DRAFT

²⁴ Note that this approach differs from the default approach in the ac.SRM tool process specification when the interest rate process is described directly by a diffusion process. Diffusion models in the ac.APM tool are explicitly approximated by discrete trees as described in this guide, for example, by the Black-Scholes equations for risk-neutral probabilities. For single-asset case, the tool provides an option to specify the asset price process explicitly as a diffusion process and the tool performs automatically the approximation of the diffusion with the binomial Markov tree.

Appendix D Derivative Markets Overview

This section provides a brief description of how the derivative markets operate by summarizing the content provided in [5] and [7] as well as other sources.

D.1 Risk transfer process

When a bank transacts with a counterparty (in option, forward or spot transaction), the bank will hedge itself in the interbank market with respective product. With the “risk transfer” approach, the transaction price offered to the counterparty is set equal to the mid market price plus pre-agreed risk transfer (execution) spread. The spread premium is added to compensate the bank for the risk taken by the bank to clear the transaction in the market. The mid market price is estimated as of the transaction execution time.

The “clearing” risk is present due to the following reasons:

1. **Execution time.** It may take significant amount of time to clear the transaction.²⁵ Spot markets can be very volatile. The mid market spot rate moves on a second-by-second basis.
2. **Market liquidity.** Liquidity of the FX markets differs by the currency pair. For example, execution of the GBP/CAD spot trade may require access to both GBP/USD and USD/CAD spot markets. Implied volatility (option) markets are typically not liquid. Transaction timing should be targeted to high market liquidity to reduce the transaction costs.²⁶ Banks use real-time data and analytics technology to screen for market liquidity conditions and market activity and to identify the time for transaction execution.
3. **Transaction size.** Large transactions have the ability to significantly move the market. **Confidentiality**²⁷ in large transaction execution is extremely important.²⁸ A large transaction can be executed either as a single trade or tranced in multiple trades. A spread premium in tranced transactions is smaller but there is a larger execution window and, therefore, a larger risk of spot rates movements.

Typical risk-transfer premium spreads are summarized below.

1. FX spot execution premium: 20-100bps.
2. FX forward execution premium: 1-5bps.
3. FX option execution premium: 1-3% of implied volatility.

Market clearing and settlement is processed by the Depository Trust & Clearing Corporation (**DTCC**), which performs the exchange of securities on behalf of buyers and sellers and functions as a central securities depository by providing central custody of securities. DTCC was established in 1999 as a holding company to combine The Depository Trust Company (DTC) and National Securities Clearing Corporation (NSCC).

²⁵ A two billion FX deal for major currencies can be executed within one day.

²⁶ For example, a GBP/CAD spot trade should be executed when both European and North American markets are open.

²⁷ Confidentiality is key to ensuring smooth execution and limiting information leakage.

²⁸ Options are executed in trades of less than C\$50mln face value per trade in the interbank market with approximately a dozen of dealers. Therefore, large option trade transactions are very difficult to execute.

DTCC automates, centralizes, standardizes, and streamlines processes in the capital markets. Through its subsidiaries, DTCC provides clearance, settlement, and information services for equities, corporate and municipal bonds, unit investment trusts, government and mortgage-backed securities, money market instruments, and **over-the-counter derivatives**.²⁹ DTCC is the highest financial value processor in the world. DTCC operates facilities in the New York metropolitan area, and at multiple locations in and outside the United States.

D.2 Early termination of hedging contracts

EY also reviewed the ISDA Master Agreement. As stated above, the ISDA Master Agreement is a standardized agreement which governs over-the-counter derivative transactions. The rail residual reimbursement transaction could be effectively viewed as a derivative transaction and the terms outlined by the ISDA Master Agreement could be considered as being comparable to arm's-length terms. The ISDA Master Agreement states the following:

- ▶ An additional termination even is specified pursuant to the following: If any "Additional Termination Event" is specified in the Schedule or any Confirmation as applying the occurrence of such event (and, in such even the Affected party or Affected Parties shall be as specified for such Additional Termination Event in the Schedule or such Confirmation)."
- ▶ In the event of early termination: "...an amount will be payable equal to the Non-Terminating Party's Loss in respect of this Agreement. If that amount is a positive number, the Terminating Party will pay it to the Non-Terminating Party; if it is a negative number, the Non-Terminating Party will pay the absolute value of that amount to the Terminating Party."

EY identified a United States Tax Court Report (Report) titled "R.V.I. Guaranty Co., Ltd. and Subsidiaries, Petitioner v. Commissioner of Internal Revenue, Respondent" filed on 21 September 2015. A high-level summary of the report is outlined below:

1. RVI, is a residual value insurance provider for vehicles, commercial real-estate and commercial equipment. The Report outlines a court case involving RVI.
2. A portion of the Report outlines many of the terms that are typical to residual value insurance policies.
3. The Report states the following key items:
 - ▶ "[Residual value insurance] policies typically called for a single premium payable at inception of the contract. The premiums charged depended on how much risk [the insurer] assumed, i.e., on the magnitude of the gap between the expected residual value and the insured value."
 - ▶ "For [the insurer] to have a liability under a contract, the insured had to meet various conditions precedent, e.g., pay the premium, having an ownership interest in the covered property..."
 - ▶ If premiums are to be taxed upon receipt, they are to be treated as earned upon receipt.
 - ▶ "[Residual value insurance] policies call for nonrefundable premiums..." "An RVI policy will pay out, if at all, on upon lease termination. In certain circumstances – for example, if inflation develops during a long-term lease – the lessor may become confident that the residual value of his leased asset will exceed its insured value at lease termination. To prevent the insured from taking a self-serving "wait and see" attitude in this setting [the lessor] may rationally choose to disallow premium refunds upon mid-stream policy cancelations.

²⁹ https://en.wikipedia.org/wiki/Depository_Trust_%26_Clearing_Corporation.

- ▶ “The IRS has recognized that there are other types of insurance, such as surety insurance, for which the policy may be made noncancellable and for which the premium therefore is non-refundable.”

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Appendix E Examples

This section discusses some real-life hedging projects.

E.1 Deal contingent FX hedging

Suppose that a company (**Parent**) is acquiring a company (**Target**) in a foreign jurisdiction using foreign currency. Acquisition hedging is performed when there is the duration between the acquisition announcement (and price setting) date and the deal closure date is material (at least several months). The Parent is exposed in this case to FX risk when the fund for the acquisition are raised in one currency (e.g. \$C) and acquisition price is set in a different currency (e.g. \$GBP). The increased FX risk exposure may be due to material political changes that are expected to happen prior to the deal closure (e.g. Brexit).

The objective of the acquisition hedging strategy is to protect against the following risks:

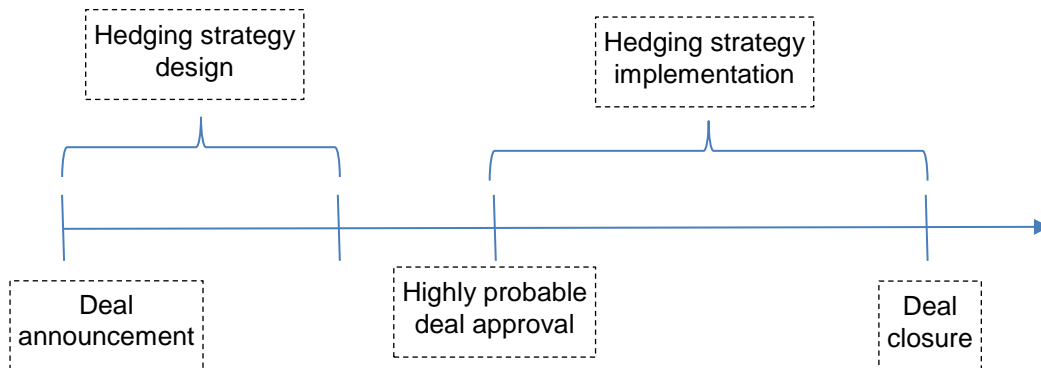
1. Hedging purchase price (**PP**)
2. Hedging book value (**BV**) accounts

This section presents a PP hedging example. The BV hedging example is presented in the next section.

E.1.1 FX hedging timing

The hedging timing is illustrated in the following diagram.

Exhibit E.1 FX hedging strategy timing



E.1.2 FX hedging strategy design

General hedging considerations are summarized in the exhibit below.

Exhibit E.2 General FX hedging strategy considerations

Strategy	Description
Acquisition financing	
Unwind existing hedges	

Strategy	Description
Deal contingent (DC) FX forwards	
FX call option	
FX forward	

A specific feature of acquisition hedging is that the following uncertainties need to be taken into account when designing a hedging strategy:

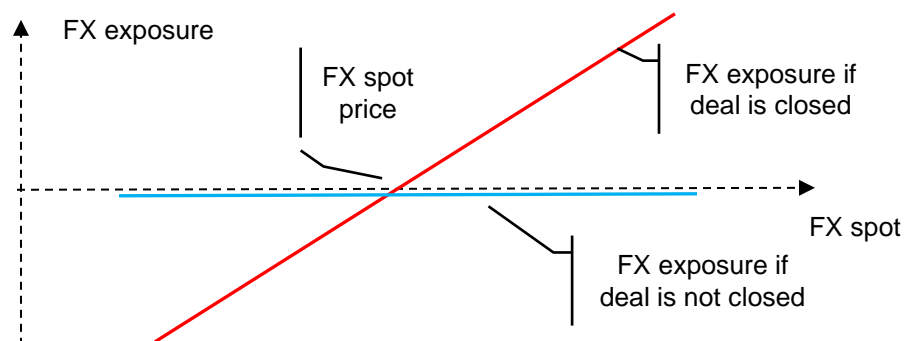
1. Uncertainty in the deal **closure**
2. Uncertainty in the deal **timing**

Then the deal timing uncertainty can be managed by **rolling over** the hedging contract if the deal is still not closed on the hedging contract termination date.

Deal uncertainty is more difficult to hedge and requires an OTC hedging contract with the bank. The problem raised by the deal uncertainty is that a forward contract provides a perfect hedge in the event if a deal closes but creates FX exposure to the Parent if the deal does not go through. Call option resolves partially the deal closure uncertainty since it hedges only the downside risk and regardless of whether the deal closes or not the Parent either receives the positive gain (in the upside scenario) or zero gain (in the upside scenario). However, because of the one-side exposure, the call options are typically expensive to use in hedging. Call option price however can be used as a reference price (upper bound price) for the DC forward since in the DC forward Parent does not have the positive gain in the upside scenario.

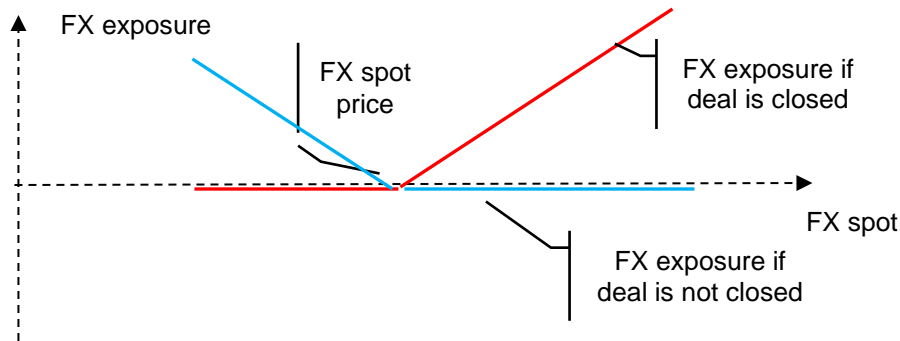
The exposure to the FX risk in the acquisition transaction is illustrated in the diagram below.

Exhibit E.3 FX exposure in the deal contingent transaction



From the Worst Loss risk measure perspective, the optimal hedging strategy which minimizes the risk and is not deal contingent is the FX put option with the ATM FX spot price (strike price is set to current FX spot price). Exposure in the deal contingent transaction hedged by the ATM put option is illustrated in the diagram below.

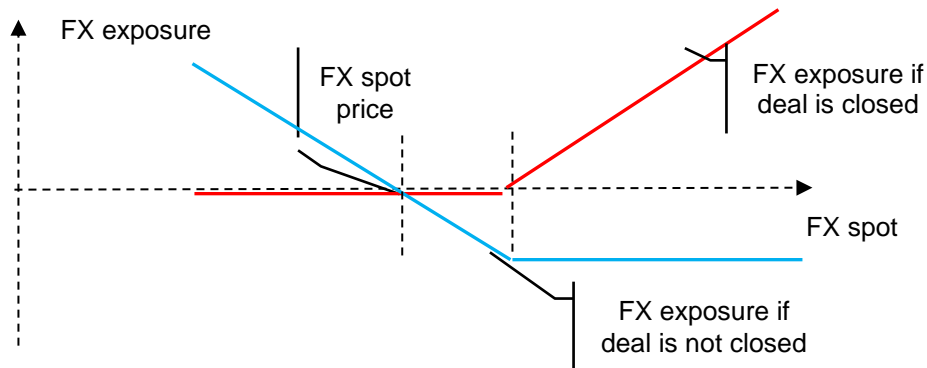
Exhibit E.4 FX exposure in the deal contingent transaction hedged by the ATM put option



The blue line, which shows the exposure in the case if the deal is not closed, also represents the payoff in the put option transaction. Under the hedging strategy, the minimum payoff with respect to deal closed or not closed contingencies is zero for any FX spot price. Therefore, any alternative hedging strategy will only increase the ES risk measure. (Suppose, for example, that the strike price of the put option is set above the spot price $K > S$ to reduce the price of the put option. Then the exposure equal to $S - K$ is created for downward movement in the spot price. The decrease in the put option price does not offset the created $S - K$ exposure.

If risk measure takes into account the probabilities of the deal closure and the probability that the deal will be closed is high, then the hedging costs can be reduced, for example, by selling the bull call spread. The exposure in this case is illustrated in the diagram below.

Exhibit E.5 FX exposure in the deal contingent transaction hedged by the ATM put option



The bull call spread creates exposure when the deal is not closed and reduces potential upside gain when the deal is not closed. On the other hand, it allows to reduce the cost of the hedging strategy. If the probability of the deal closure is assessed as high, then the exposure can be viewed as a reasonable price for the reduction in the hedging costs.

E.1.3 Valuation of a DC forward

As discussed above, a put option is the optimal strategy from the Worst Loss risk measure minimization. However, the put option is an expensive hedging strategy and the Bank can agree to sell a DC forward to hedge the FX price for a fraction of the put option price.

E.2 Book value hedging

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Appendix F Technical Comments

In this section we provide some technical integration formulas used in the models described in this guide.

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Appendix G References

List of literature references used in the notes is provided below

G.1 Lease valuation and hedging

- [1] Literature references on lease valuation and hedging models.
- [2] David C. Rode, Paul S. Fishbeck, Steve R. Dean, "Residual Risk and the Valuation of Leases under Uncertainty and Limited Information", Carnegie Mellon Electricity Industry Center, CEIC Working Paper 02-02, 2002³⁰
- [3] Katharina Nau, "An Empirical Analysis of Residual Value Risk in Automotive Lease Contracts", Dissertation, Hohenheim University, Institute of Financial Management, 2012³¹
- [4] Sylvain Prado, "Hedging Residual Value Risk Using Derivatives", working paper, UPX, 2009-31, December 2008³²

G.2 Derivative pricing

- [5] CFA, Level I, Volume 6, "Derivative and Alternative Investments", 2013
- [6] DerivativeEngines.com, Real Time Option Calculator, <http://www.derivativeengines.somee.com/index-3.asp>
- [7] Financial Risk Manager (FRM), Part I, "Financial Markets and Products", 2016
- [8] Financial Risk Manager (FRM), Part I, Book 4, "Valuation and Risk Models", 2014

³⁰ The paper is also published as Rode, D., P. Fischbeck, and S. Dean. "Residual Risk and the Valuation of Leases under Uncertainty and Limited Information". *Journal of Structured and Project Finance* 7:4 (2002): 37-49. Online link to the paper: <https://pdfs.semanticscholar.org/c517/301420d394a05698a876adfd74e541504758.pdf>

³¹ <https://core.ac.uk/download/pdf/56707500.pdf>

³² Online link: https://www.researchgate.net/publication/254406042_Hedging_residual_value_risk_using_derivatives